

CSC 370 - Computer Organization and Architecture

Lecture 3: Quine McCluskey Algorithm

Why Use the Quine McCluskey Algorithm?

- While Karnaugh maps provide an easy method for simplifying the functions that we have seen so far, they do not work well when there are more than six independent variables.
- One common method is the Quine McCluskey Algorithm, which makes extensive use of the axiom $\mathbf{xy' + xy = x}$ to simplify terms.

The Basic Steps in the Algorithm

1. Write the function F as the sum of minterms
2. By using the rule $\mathbf{xy' + xy = x}$, eliminate as many terms as possible to produce the prime implicants.
3. Use a prime implicant charts to eliminate redundant terms.

Rewriting and Combining Minterms

- We start out by rewriting minterms as binary numbers:

E.g.,

$w'x'yz$ (m_3) becomes 0011

$w'x'y'z$ (m_1) 0001

These combine to form 00-1 $w'x'z$

- We can only combine them if they differ by *exactly 1 bit*.

Example: Simplifying $\Sigma(0,4,5,7,8,11,12,15)$

- We write the minterms in binary form, grouping them by the number of ones in the binary form.

m_0	<u>0000</u>	0 1's
m_4	<u>0100</u>	1 1's
m_8	<u>1000</u>	
m_5	<u>0101</u>	2 1's
m_{12}	<u>1100</u>	
m_7	<u>0111</u>	3 1's
m_{11}	<u>1011</u>	
m_{15}	<u>1111</u>	4 1's

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We now look for minterms whose binary form differs with only one bit:

m_0	<u>0000</u>	←	(0,4)	0-00
m_4	<u>0100</u>	←		
m_8	<u>1000</u>			
m_5	<u>0101</u>			
m_{12}	<u>1100</u>			
m_7	<u>0111</u>			
m_{11}	<u>1011</u>			
m_{15}	<u>1111</u>			

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We check the minterms for which there is a simplification:

m_0	$\sqrt{0000}$	(0,4) 0-00
m_4	$\sqrt{0100}$	
m_8	$\underline{1000}$	
m_5	0101	
m_{12}	$\underline{1100}$	
m_7	0111	
m_{11}	$\underline{1011}$	
m_{15}	1111	

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4) 0-00
m_4	$\sqrt{0100}$	(0,8) -000
m_8	$\sqrt{1000}$	
m_5	$\underline{0101}$	
m_{12}	$\underline{1100}$	
m_7	0111	
m_{11}	$\underline{1011}$	
m_{15}	1111	

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4,5)	010-
m_5	$\sqrt{0101}$		
m_{12}	$\underline{1100}$		
m_7	0111		
m_{11}	$\underline{1011}$		
m_{15}	1111		

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4,5)	010-
m_5	$\sqrt{0101}$	(4,12)	-100
m_{12}	$\sqrt{1100}$		
m_7	0111		
m_{11}	$\underline{1011}$		
m_{15}	1111		

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4, 5)	010-
m_5	$\sqrt{0101}$	(4,12)	-100
m_{12}	$\sqrt{1100}$	(8,12)	1-00
m_7	0111		
m_{11}	1011		
m_{15}	1111		

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4, 5)	010-
m_5	$\sqrt{0101}$	(4,12)	-100
m_{12}	$\sqrt{1100}$	(8,12)	1-00
m_7	$\sqrt{0111}$	(5, 7)	01-1
m_{11}	1011		
m_{15}	1111		

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4, 5)	010-
m_5	$\sqrt{0101}$	(4,12)	-100
m_{12}	$\sqrt{1100}$	(8,12)	1-00
m_7	$\sqrt{0111}$	(5, 7)	01-1
m_{11}	$\sqrt{1011}$	(7, 15)	-111
m_{15}	$\sqrt{1111}$		

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

m_0	$\sqrt{0000}$	(0,4)	0-00
m_4	$\sqrt{0100}$	(0,8)	-000
m_8	$\sqrt{1000}$	(4, 5)	010-
m_5	$\sqrt{0101}$	(4,12)	-100
m_{12}	$\sqrt{1100}$	(8,12)	1-00
m_7	$\sqrt{0111}$	(5, 7)	01-1
m_{11}	$\sqrt{1011}$	(7, 15)	-111
m_{15}	$\sqrt{1111}$	(11, 15)	1-11

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We now do the same thing to the newly simplified terms:

m_0	$\sqrt{0000}$	$\sqrt{(0,4)}$	0-00	\leftarrow	(0,4,8,12)	--00
m_4	$\sqrt{0100}$	(0,8)	<u>-000</u>			
m_8	$\sqrt{1000}$	(4,5)	010-			
m_5	$\sqrt{0101}$	(4,12)	-100			
m_{12}	$\sqrt{1100}$	$\sqrt{(8,12)}$	<u>1-00</u>	\swarrow		
m_7	$\sqrt{0111}$	(5,7)	<u>01-1</u>			
m_{11}	$\sqrt{1011}$	(7,15)	-111			
m_{15}	$\sqrt{1111}$	(11,15)	1-11			

Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- These new terms may produce simplified terms that are copies of each other:

m_0	$\sqrt{0000}$	$\sqrt{(0,4)}$	0-00		(0,4,8,12)	--00
m_4	$\sqrt{0100}$	$\sqrt{(0,8)}$	<u>-000</u>	\leftarrow	(0,8,4,12)	--00
m_8	$\sqrt{1000}$	(4,5)	010-			
m_5	$\sqrt{0101}$	$\sqrt{(4,12)}$	-100	\swarrow		
m_{12}	$\sqrt{1100}$	$\sqrt{(8,12)}$	<u>1-00</u>			
m_7	$\sqrt{0111}$	(5,7)	<u>01-1</u>			
m_{11}	$\sqrt{1011}$	(7,15)	-111			
m_{15}	$\sqrt{1111}$	(11,15)	1-11			

The Terms Left In $\Sigma(0,4,5,7,8,11,12,15)$

- We now have:

(0,4,8,12) --00

~~(0,8,4,12) --00~~ Duplicate term

(4, 5) 010-

(5, 7) 01-1

(7, 15) -111

(11, 15) 1-11

$$F = y'z' + w'xy' + w'xz + xyz + wyz$$

There is still some duplicate even after finding the prime implicants. We need the Prime Implicant Table to eliminate the unnecessary terms

Prime Implicant Table

PI	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						X		X

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						X		X

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						X		X

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						X		X

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						X		X

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	X					
$w'xz$	01-1			X	X				
xyz	-111				X				X
wyz	1-11						x		x

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	X					
$w'xz$	01-1			X	X				
xyz	-111				X				x
wyz	1-11						x		x

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
xyz	-111				x				x
wyz	1-11						x		x

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
xyz	-111				x				x
wyz	1-11						x		x

Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
xyz	-111				x				x
wyz	1-11						x		x

$$F = y'z' + w'xz + wyz$$

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0 0000 0 1's

m_1 0001 1 1's

m_2 0010

m_8 1000

m_5 0101 2 1's

m_6 0110

m_9 1001

m_{10} 1010

m_7 0111 3 1's

m_{14} 1110

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	←	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	←		
m_2	0010			
m_8	$\frac{1000}{\phantom{\sqrt{}}}$			
m_5	0101			
m_6	0110			
m_9	1001			
m_{10}	$\frac{1010}{\phantom{\sqrt{}}}$			
m_7	0111			
m_{14}	1110			

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	←	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	←	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	←		
m_8	$\frac{1000}{\phantom{\sqrt{}}}$			
m_5	0101			
m_6	0110			
m_9	1001			
m_{10}	$\frac{1010}{\phantom{\sqrt{}}}$			
m_7	0111			
m_{14}	1110			

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	$(0,1)$	000-
m_1	$\frac{\sqrt{0001}}$	$(0,2)$	00-0
m_2	$\frac{\sqrt{0010}}$	$(0,8)$	-000
m_8	$\frac{\sqrt{1000}}$		
m_5	0101		
m_6	0110		
m_9	1001		
m_{10}	$\frac{1010}$		
m_7	0111		
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	$(0,1)$	000-
m_1	$\frac{\sqrt{0001}}$	$(0,2)$	00-0
m_2	$\frac{\sqrt{0010}}$	$(0,8)$	-000
m_8	$\frac{\sqrt{1000}}$	$(1,5)$	0-01
m_5	$\frac{\sqrt{0101}}$		
m_6	0110		
m_9	1001		
m_{10}	$\frac{1010}$		
m_7	0111		
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	0110		
m_9	$\sqrt{1001}$		
m_{10}	$\frac{1010}{}$		
m_7	0111		
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	$\frac{\sqrt{0110}}$	(2,6)	0-10
m_9	$\frac{\sqrt{1001}}$	(2,10)	-010
m_{10}	$\frac{\sqrt{1010}}$		
m_7	0111		
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	$\frac{\sqrt{0110}}$	(2,6)	0-10
m_9	$\frac{\sqrt{1001}}$	(2,10)	-010
m_{10}	$\frac{\sqrt{1010}}$	(8,9)	100-
m_7	0111		
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	$\frac{\sqrt{0110}}$	(2,6)	0-10
m_9	$\frac{\sqrt{1001}}$	(2,10)	-010
m_{10}	$\frac{\sqrt{1010}}$	(8,9)	100-
m_7	0111	(8,10)	10-0
m_{14}	1110		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	$\frac{\sqrt{0110}}$	(2,6)	0-10
m_9	$\frac{\sqrt{1001}}$	(2,10)	-010
m_{10}	$\frac{\sqrt{1010}}$	(8,9)	100-
m_7	$\frac{\sqrt{0111}}$	(8,10)	10-0
m_{14}	1110	(5,7)	01-1

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\frac{\sqrt{0000}}$	(0,1)	000-
m_1	$\frac{\sqrt{0001}}$	(0,2)	00-0
m_2	$\frac{\sqrt{0010}}$	(0,8)	-000
m_8	$\frac{\sqrt{1000}}$	(1,5)	0-01
m_5	$\frac{\sqrt{0101}}$	(1,9)	-001
m_6	$\frac{\sqrt{0110}}$	(2,6)	0-10
m_9	$\frac{\sqrt{1001}}$	(2,10)	-010
m_{10}	$\frac{\sqrt{1010}}$	(8,9)	100-
m_7	$\frac{\sqrt{0111}}$	(8,10)	10-0
m_{14}	1110	(5,7)	01-1
		(6,7)	011-

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

m_0	$\sqrt{0000}$	(0,1)	000-
m_1	$\sqrt{0001}$	(0,2)	00-0
m_2	$\sqrt{0010}$	(0,8)	-000
m_8	$\sqrt{1000}$	(1,5)	0-01
m_5	$\sqrt{0101}$	(1,9)	-001
m_6	$\sqrt{0110}$	(2,6)	0-10
m_9	$\sqrt{1001}$	(2,10)	-010
m_{10}	$\sqrt{1010}$	(8,9)	100-
m_7	$\sqrt{0111}$	(8,10)	10-0
m_{14}	$\sqrt{1110}$	(5,7)	01-1
		(6,7)	011-
		(6,14)	-110
		(10,14)	1-10

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	$(0,1,8,9)$	-00-
(0,2)	00-0		
(0,8)	-000		
(1,5)	0-01		
(1,9)	-001		
(2,6)	0-10		
(2,10)	-010		
$\sqrt{(8,9)}$	100-		
(8,10)	10-0		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	← (0,2,8,10)	-0-0
(0,8)	<u>-000</u>		
(1,5)	0-01		
(1,9)	-001		
(2,6)	0-10		
(2,10)	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	← (0,8,1,9)	-00-
(1,5)	0-01		
$\sqrt{(1,9)}$	-001		
(2,6)	0-10		
(2,10)	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	-000	(0,8,1,9)	-00-
(1,5)	<u>0-01</u>	(0,8,2,10)	-0-0
$\sqrt{(1,9)}$	-001		
(2,6)	0-10		
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	-000	(0,8,1,9)	-00-
(1,5)	<u>0-01</u>	(0,8,2,10)	-0-0
$\sqrt{(1,9)}$	-001	(2,6,10,14)	--10
$\sqrt{(2,6)}$	0-10		
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
$\sqrt{(10,14)}$	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	(0,8,1,9)	-00-
(1,5)	0-01	(0,8,2,10)	-0-0
$\sqrt{(1,9)}$	-001	(2,6,10,14)	--10
$\sqrt{(2,6)}$	0-10	(2,10,6,14)	--10
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
$\sqrt{(6,14)}$	-110		
$\sqrt{(10,14)}$	1-10		

Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

- These are the terms that we have left:

(0,1,8,9)	-00-	
(0,2,8,10)	-0-0	
(0,8,1,9)	00	← <i>Redundant</i>
(0,8,2,10)	-0-0	←
(2,6,10,14)	--10	
(2,10,6,14)	--10	←
(1,5)	<u>0-01</u>	
(5,7)	01-1	
(6,7)	011-	

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
yz'	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

$$F = x'y' + yz' + w'xz$$

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

m_0	0000	0 1's
m_1	0001	1 1's
m_2	0010	
m_4	0100	
m_8	1000	
m_5	0101	2 1's
m_6	0110	
m_9	1001	
m_{12}	1100	
m_{13}	1101	3 1's
m_{14}	1110	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	←	(0, 1)	000-
$\sqrt{m_1}$	0001	←		
m_2	0010			
m_4	0100			
m_8	1000			
m_5	0101			
m_6	0110			
m_9	1001			
m_{12}	1100			
m_{13}	1101			
m_{14}	1110			

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	←	(0, 1)	000-
$\sqrt{m_1}$	0001	←	(0, 2)	00-0
$\sqrt{m_2}$	0010	←		
m_4	0100			
m_8	1000			
m_5	0101			
m_6	0110			
m_9	1001			
m_{12}	1100			
m_{13}	1101			
m_{14}	1110			

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	←	(0, 1) 000-
$\sqrt{m_1}$	0001		(0, 2) 00-0
$\sqrt{m_2}$	0010		(0, 4) 0-00
$\sqrt{m_4}$	0100	←	
m_8	1000		
m_5	0101		
m_6	0110		
m_9	1001		
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	←	(0, 1) 000-
$\sqrt{m_1}$	0001		(0, 2) 00-0
$\sqrt{m_2}$	0010		(0, 4) 0-00
$\sqrt{m_4}$	0100		(0, 8) -000
$\sqrt{m_8}$	1000	←	
m_5	0101		
m_6	0110		
m_9	1001		
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101		
m_6	0110		
m_9	1001		
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
m_6	0110		
$\sqrt{m_9}$	1001		
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001		
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001	(4, 6)	01-0
m_{12}	1100		
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001	(4, 6)	01-0
$\sqrt{m_{12}}$	1100	(4, 12)	-100
m_{13}	1101		
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001	(4, 6)	01-0
$\sqrt{m_{12}}$	1100	(4, 12)	-100
m_{13}	1101	(8, 9)	100-
m_{14}	1110		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001	(4, 6)	01-0
m_{12}	1100	(4, 12)	-100
m_{13}	1101	(8, 9)	100-
m_{14}	1110	(8, 12)	1-00

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{m_0}$	0000	(0, 1)	000-
$\sqrt{m_1}$	0001	(0, 2)	00-0
$\sqrt{m_2}$	0010	(0, 4)	0-00
$\sqrt{m_4}$	0100	(0, 8)	-000
$\sqrt{m_8}$	1000	(1, 5)	0-01
$\sqrt{m_5}$	0101	(1, 9)	-001
$\sqrt{m_6}$	0110	(2, 6)	0-10
$\sqrt{m_9}$	1001	(4, 6)	01-0
$\sqrt{m_{12}}$	1100	(4, 12)	-100
m_{13}	1101	(8, 9)	100-
m_{14}	1110	(8, 12)	1-00

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

		(0, 1)	000-
		(0, 2)	00-0
$\sqrt{m_0}$	<u>000</u>	(0, 4)	0-00
$\sqrt{m_1}$	<u>0001</u>	(0, 8)	-000
$\sqrt{m_2}$	0010	(1, 5)	0-01
$\sqrt{m_4}$	0100	(1, 9)	-001
$\sqrt{m_8}$	<u>1000</u>	(2, 6)	0-10
$\sqrt{m_5}$	<u>0101</u>	(4, 6)	01-0
$\sqrt{m_6}$	0110	(4, 12)	-100
$\sqrt{m_9}$	1001	(8, 9)	100-
$\sqrt{m_{12}}$	<u>1100</u>	(8, 12)	1-00
$\sqrt{m_{13}}$	1101	(5, 13)	-101
$\sqrt{m_{14}}$	1110	(6, 14)	-110

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

		(0, 1)	000-
		(0, 2)	00-0
$\sqrt{m_0}$	<u>000</u>	(0, 4)	0-00
$\sqrt{m_1}$	<u>0001</u>	(0, 8)	-000
$\sqrt{m_2}$	0010	(1, 5)	0-01
$\sqrt{m_4}$	0100	(1, 9)	-001
$\sqrt{m_8}$	<u>1000</u>	(2, 6)	0-10
$\sqrt{m_5}$	<u>0101</u>	(4, 6)	01-0
$\sqrt{m_6}$	0110	(4, 12)	-100
$\sqrt{m_9}$	1001	(8, 9)	100-
$\sqrt{m_{12}}$	<u>1100</u>	(8, 12)	1-00
$\sqrt{m_{13}}$	1101	(5, 13)	-101
$\sqrt{m_{14}}$	1110	(6, 14)	-110
		(9, 13)	1-01

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

		(0, 1)	000-
		(0, 2)	00-0
$\sqrt{m_0}$	<u>000</u>	(0, 4)	0-00
$\sqrt{m_1}$	<u>0001</u>	(0, 8)	-000
$\sqrt{m_2}$	0010	(1, 5)	0-01
$\sqrt{m_4}$	0100	(1, 9)	-001
$\sqrt{m_8}$	<u>1000</u>	(2, 6)	0-10
$\sqrt{m_5}$	<u>0101</u>	(4, 6)	01-0
$\sqrt{m_6}$	0110	(4, 12)	-100
$\sqrt{m_9}$	1001	(8, 9)	100-
$\sqrt{m_{12}}$	<u>1100</u>	(8, 12)	1-00
$\sqrt{m_{13}}$	<u>1101</u>	(5, 13)	-101
$\sqrt{m_{14}}$	1110	(6, 14)	-110
		(9, 13)	1-01
		(12, 13)	110-

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

		(0, 1)	000-
		(0, 2)	00-0
$\sqrt{m_0}$	<u>000</u>	(0, 4)	0-00
$\sqrt{m_1}$	<u>0001</u>	(0, 8)	-000
$\sqrt{m_2}$	0010	(1, 5)	0-01
$\sqrt{m_4}$	0100	(1, 9)	-001
$\sqrt{m_8}$	<u>1000</u>	(2, 6)	0-10
$\sqrt{m_5}$	<u>0101</u>	(4, 6)	01-0
$\sqrt{m_6}$	0110	(4, 12)	-100
$\sqrt{m_9}$	1001	(8, 9)	100-
$\sqrt{m_{12}}$	<u>1100</u>	(8, 12)	1-00
$\sqrt{m_{13}}$	<u>1101</u>	(5, 13)	-101
$\sqrt{m_{14}}$	1110	(6, 14)	-110
		(9, 13)	1-01
		(12, 13)	110-
		(12, 14)	11-0

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-	$(0, 1, 8, 9)$	-00-
(0, 2)	00-0		
(0, 4)	0-00		
(0, 8)	-000		
(1, 5)	0-01		
(1, 9)	-001		
(2, 6)	0-10		
(4, 6)	01-0		
(4, 12)	-100		
$\sqrt{(8, 9)}$	100-		
(8, 12)	1-00		
(5, 13)	-101		
(6, 14)	-110		
(9, 13)	1-01		
(12, 13)	110-		
(12, 14)	11-0		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-	$(0, 1, 8, 9)$	-00-
$\sqrt{(0, 2)}$	00-0	$(0, 2, 4, 6)$	0--0
(0, 4)	0-00		
(0, 8)	-000		
(1, 5)	0-01		
(1, 9)	-001		
(2, 6)	0-10		
$\sqrt{(4, 6)}$	01-0		
(4, 12)	-100		
$\sqrt{(8, 9)}$	100-		
(8, 12)	1-00		
(5, 13)	-101		
(6, 14)	-110		
(9, 13)	1-01		
(12, 13)	110-		
(12, 14)	11-0		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-		$(0, 1, 8, 9)$	-00-
$\sqrt{(0, 2)}$	00-0		$(0, 2, 4, 6)$	0--0
$\sqrt{(0, 4)}$	0-00	←	$(0, 4, 2, 6)$	0--0
$(0, 8)$	-000			
$(1, 5)$	0-01			
$(1, 9)$	-001			
$\sqrt{(2, 6)}$	0-10	↙		
$\sqrt{(4, 6)}$	01-0			
$(4, 12)$	-100			
$\sqrt{(8, 9)}$	100-			
$(8, 12)$	1-00			
$(5, 13)$	-101			
$(6, 14)$	-110			
$(9, 13)$	1-01			
$(12, 13)$	110-			
$(12, 14)$	11-0			

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-		$(0, 1, 8, 9)$	-00-
$\sqrt{(0, 2)}$	00-0		$(0, 2, 4, 6)$	0--0
$\sqrt{(0, 4)}$	0-00	←	$(0, 4, 2, 6)$	0--0
$(0, 8)$	-000		$(0, 4, 8, 12)$	--00
$(1, 5)$	0-01			
$(1, 9)$	-001			
$\sqrt{(2, 6)}$	0-10			
$\sqrt{(4, 6)}$	01-0			
$(4, 12)$	-100			
$\sqrt{(8, 9)}$	100-			
$\sqrt{(8, 12)}$	1-00	↙		
$(5, 13)$	-101			
$(6, 14)$	-110			
$(9, 13)$	1-01			
$(12, 13)$	110-			
$(12, 14)$	11-0			

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-	$(0, 1, 8, 9)$	-00-
$\sqrt{(0, 2)}$	00-0	$(0, 2, 4, 6)$	0--0
$\sqrt{(0, 4)}$	0-00	$(0, 4, 2, 6)$	0--0
$\sqrt{(0, 8)}$	-000	$(0, 4, 8, 12)$	--00
$(1, 5)$	<u>0-01</u>	$(0, 8, 1, 9)$	-00-
$\sqrt{(1, 9)}$	-001		
$\sqrt{(2, 6)}$	0-10		
$\sqrt{(4, 6)}$	01-0		
$(4, 12)$	-100		
$\sqrt{(8, 9)}$	100-		
$\sqrt{(8, 12)}$	<u>1-00</u>		
$(5, 13)$	-101		
$(6, 14)$	-110		
$(9, 13)$	1-01		
$(12, 13)$	110-		
$(12, 14)$	11-0		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$	000-	$(0, 1, 8, 9)$	-00-
$\sqrt{(0, 2)}$	00-0	$(0, 2, 4, 6)$	0--0
$\sqrt{(0, 4)}$	0-00	$(0, 4, 2, 6)$	0--0
$\sqrt{(0, 8)}$	-000	$(0, 4, 8, 12)$	--00
$(1, 5)$	<u>0-01</u>	$(0, 8, 1, 9)$	-00-
$\sqrt{(1, 9)}$	-001	$(0, 8, 4, 12)$	--00
$\sqrt{(2, 6)}$	0-10		
$\sqrt{(4, 6)}$	01-0		
$\sqrt{(4, 12)}$	-100		
$\sqrt{(8, 9)}$	100-		
$\sqrt{(8, 12)}$	<u>1-00</u>		
$(5, 13)$	-101		
$(6, 14)$	-110		
$(9, 13)$	1-01		
$(12, 13)$	110-		
$(12, 14)$	11-0		

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	(0, 1, 8, 9) -00-
$\sqrt{(0, 2)}$ 00-0	(0, 2, 4, 6) 0--0
$\sqrt{(0, 4)}$ 0-00	(0, 4, 2, 6) 0--0
$\sqrt{(0, 8)}$ -000	(0, 4, 8, 12) --00
$\sqrt{(1, 5)}$ 0-01	(0, 8, 1, 9) -00-
$\sqrt{(1, 9)}$ -001	(0, 8, 4, 12) --00
$\sqrt{(2, 6)}$ 0-10	(1, 5, 9, 13) --01
$\sqrt{(4, 6)}$ 01-0	
$\sqrt{(4, 12)}$ -100	
$\sqrt{(8, 9)}$ 100-	
$\sqrt{(8, 12)}$ 1-00	
(5, 13) -101	
(6, 14) -110	
$\sqrt{(9, 13)}$ 1-01	
(12, 13) 110-	
(12, 14) 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	(0, 1, 8, 9) -00-
$\sqrt{(0, 2)}$ 00-0	(0, 2, 4, 6) 0--0
$\sqrt{(0, 4)}$ 0-00	(0, 4, 2, 6) 0--0
$\sqrt{(0, 8)}$ -000	(0, 4, 8, 12) --00
$\sqrt{(1, 5)}$ 0-01	(0, 8, 1, 9) -00-
$\sqrt{(1, 9)}$ -001	(0, 8, 4, 12) --00
$\sqrt{(2, 6)}$ 0-10	(1, 5, 9, 13) --01
$\sqrt{(4, 6)}$ 01-0	(1, 9, 5, 13) --01
$\sqrt{(4, 12)}$ -100	
$\sqrt{(8, 9)}$ 100-	
$\sqrt{(8, 12)}$ 1-00	
$\sqrt{(5, 13)}$ -101	
(6, 14) -110	
$\sqrt{(9, 13)}$ 1-01	
(12, 13) 110-	
(12, 14) 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	(0, 1, 8, 9) -00-
$\sqrt{(0, 2)}$ 00-0	(0, 2, 4, 6) 0--0
$\sqrt{(0, 4)}$ 0-00	(0, 4, 2, 6) 0--0
$\sqrt{(0, 8)}$ -000	(0, 4, 8, 12) --00
$\sqrt{(1, 5)}$ 0-01	(0, 8, 1, 9) -00-
$\sqrt{(1, 9)}$ -001	(0, 8, 4, 12) --00
$\sqrt{(2, 6)}$ 0-10	(1, 5, 9, 13) --01
$\sqrt{(4, 6)}$ 01-0	(1, 9, 5, 13) --01
$\sqrt{(4, 12)}$ -100	(4, 6, 12, 14) -1-0
$\sqrt{(8, 9)}$ 100-	
$\sqrt{(8, 12)}$ 1-00	
$\sqrt{(5, 13)}$ -101	
(6, 14) -110	
$\sqrt{(9, 13)}$ 1-01	
(12, 13) 110-	
$\sqrt{(12, 14)}$ 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	(0, 1, 8, 9) -00-
$\sqrt{(0, 2)}$ 00-0	(0, 2, 4, 6) 0--0
$\sqrt{(0, 4)}$ 0-00	(0, 4, 2, 6) 0--0
$\sqrt{(0, 8)}$ -000	(0, 4, 8, 12) --00
$\sqrt{(1, 5)}$ 0-01	(0, 8, 1, 9) -00-
$\sqrt{(1, 9)}$ -001	(0, 8, 4, 12) --00
$\sqrt{(2, 6)}$ 0-10	(1, 5, 9, 13) --01
$\sqrt{(4, 6)}$ 01-0	(1, 9, 5, 13) --01
$\sqrt{(4, 12)}$ -100	(4, 6, 12, 14) -1-0
$\sqrt{(8, 9)}$ 100-	(4, 12, 6, 14) -1-0
$\sqrt{(8, 12)}$ 1-00	
$\sqrt{(5, 13)}$ -101	
$\sqrt{(6, 14)}$ -110	
$\sqrt{(9, 13)}$ 1-01	
(12, 13) 110-	
$\sqrt{(12, 14)}$ 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	$(0, 1, 8, 9)$ -00-
$\sqrt{(0, 2)}$ 00-0	$(0, 2, 4, 6)$ 0--0
$\sqrt{(0, 4)}$ 0-00	$(0, 4, 2, 6)$ 0--0
$\sqrt{(0, 8)}$ -000	$(0, 4, 8, 12)$ --00
$\sqrt{(1, 5)}$ 0-01	$(0, 8, 1, 9)$ -00-
$\sqrt{(1, 9)}$ -001	$(0, 8, 4, 12)$ --00
$\sqrt{(2, 6)}$ 0-10	$(1, 5, 9, 13)$ --01
$\sqrt{(4, 6)}$ 01-0	$(1, 9, 5, 13)$ --01
$\sqrt{(4, 12)}$ -100	$(4, 6, 12, 14)$ -1-0
$\sqrt{(8, 9)}$ 100-	$(4, 12, 6, 14)$ -1-0
$\sqrt{(8, 12)}$ 1-00	$(8, 9, 12, 13)$ 1-0-
$\sqrt{(5, 13)}$ -101	
$\sqrt{(6, 14)}$ -110	
$\sqrt{(9, 13)}$ 1-01	
$\sqrt{(12, 13)}$ 110-	
$\sqrt{(12, 14)}$ 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

$\sqrt{(0, 1)}$ 000-	$(0, 1, 8, 9)$ -00-
$\sqrt{(0, 2)}$ 00-0	$(0, 2, 4, 6)$ 0--0
$\sqrt{(0, 4)}$ 0-00	$(0, 4, 2, 6)$ 0--0
$\sqrt{(0, 8)}$ -000	$(0, 4, 8, 12)$ --00
$\sqrt{(1, 5)}$ 0-01	$(0, 8, 1, 9)$ -00-
$\sqrt{(1, 9)}$ -001	$(0, 8, 4, 12)$ --00
$\sqrt{(2, 6)}$ 0-10	$(1, 5, 9, 13)$ --01
$\sqrt{(4, 6)}$ 01-0	$(1, 9, 5, 13)$ --01
$\sqrt{(4, 12)}$ -100	$(4, 6, 12, 14)$ -1-0
$\sqrt{(8, 9)}$ 100-	$(4, 12, 6, 14)$ -1-0
$\sqrt{(8, 12)}$ 1-00	$(8, 9, 12, 13)$ 1-0-
$\sqrt{(5, 13)}$ -101	$(8, 12, 9, 13)$ 1-0-
$\sqrt{(6, 14)}$ -110	
$\sqrt{(9, 13)}$ 1-01	
$\sqrt{(12, 13)}$ 110-	
$\sqrt{(12, 14)}$ 11-0	

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

(0, 1, 8, 9) -00-	
(0, 2, 4, 6) 0--0	
(0, 4, 2, 6) 0--0	←
(0, 4, 8, 12) --00	
(0, 8, 1, 9) --00	←
(0, 8, 4, 12) --00	←
(1, 5, 9, 13) --01	
(1, 9, 5, 13) --01	←
(4, 6, 12, 14) -1-0	
(4, 12, 6, 14) -1-0	←
(8, 9, 12, 13) 1-0-	
(8, 12, 9, 13) 1-0-	←

Duplicate terms

Example: $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

(0, 1, 8, 9) -00-	(0, 4, 8, 12, 1, 5, 9, 13) --0-
(0, 2, 4, 6) 0--0	
√(0, 4, 8, 12) --00	
√(1, 5, 9, 13) --01	
(4, 6, 12, 14) -1-0	
(8, 9, 12, 13) 1-0-	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X					X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X					X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X		X			X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X		X			X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X					X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	X	X		X	X		X	X	X	X	
x'y'	-00-	X	X					X	X			
w'z'	0--0	X		X	X		X					
xz'					X		X			X		X
wy'	1-0-							X	X	X	X	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	x	x		x	x		x	x	x	x	
x'y'	-00-	x	x					x	x			
w'z'	0--0	x		x	x		x					
xz'					x		x			x		x
wy'	1-0-							x	x	x	x	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	x	x		x	x		x	x	x	x	
x'y'	-00-	x	x					x	x			
w'z'	0--0	x		x	x		x					
xz'					x		x			x		x
wy'	1-0-							x	x	x	x	

Prime Implicants of $\Sigma(0,1, 2, 4, 5, 6, 8, 9,12, 13,14)$

PI	Numeri c	m ₀	m ₁	m ₂	m ₄	m ₅	m ₆	m ₈	m ₉	m ₁₂	m ₁₃	m ₁₄
y'	--0-	x	x		x	x		x	x	x	x	
x'y'	-00-	x	x					x	x			
w'z'	0--0	x		x	x		x					
xz'					x		x			x		x
wy'	1-0-							x	x	x	x	

$$F = y' + w'z' + xz'$$