Mapping Boolean Functions

- Boolean expressions can be fairly complex.
  - This leads to overly-complex digital circuits.
  - This necessitates simplification of Boolean expressions.
- These expressions can become too complex for simplification by Boolean algebra.
- The use of maps such as Karnaugh maps makes it much easier to simplify such expressions.
Two-variable maps

Each square represents a minterm.

Representing 2-Variable Functions

\[ F = xy = x'y + xy' + xy \]
Three-variable maps

- With only two dimensions on the page, we need to graph more than one dimension together whenever we go beyond two-variable maps.
- We arrange the minterms in an order that resembles Gray codes, where only one bit varies between adjacent squares.
- This allows us to recognize simpler terms quickly.
Simplifying 3-Variable Expressions

\[ F = x'y'z + x'yz' + xy'z' + xy'z \]
\[ = xy' + x'y \]

\[ F = x'y'z + xy'z' + xy + xyz' \]
\[ = xz' + yz \]
Simplifying 3-Variable Expressions (continued)

\( F = A'C + A'B + AB'C + BC \)
\[ = C + A'B \]

Simplifying 3-Variable Expressions (continued)

\( F = \Sigma (0, 2, 4, 5, 6) \)
\[ = z' + xy' \]
Four-Variable Maps

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<th>m₀</th>
<th>m₁</th>
<th>m₃</th>
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F = Σ(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'

Simplifying 4-variable Maps
Simplifying 4-variable Maps (continued)

\[ F = \Sigma(1, 3, 4, 5, 9, 11, 12, 13, 14, 15) = wx + xy' + x'z \]

\[ F = A'B'C' + B'CD + A'BCD + AB'C' = B'D + B'C' + A'CD \]
Don’t Care Conditions

• Until now, all the spaces on a Karnaugh map indicates where the function has a value of “1” or “0” (assumed by the square being left empty).
• Sometimes we don’t care what value the square holds – it is irrelevant.
• We can’t leave it blank (assumed to be 0), 0 or 1, so we mark it with an “X” to indicate that we don’t care about its value.

Don’t-Care Conditions – An Example

\[ F = \Sigma(1,3,7,11,15) \]
\[ d = \Sigma(0, 2, 5) \]
\[ F = w'z + yz = z (w' + y) \]
Don’t-Care Conditions – An Example

\[ F = \Sigma(1,2,3,5,7) \]
\[ d = \Sigma(10,11,12,13,14,15) \]
\[ F = w'z + x'y \]

5-Variable Maps

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Product of Maxterms

- Normally, we express Boolean function as a sum of minterms, e.g.,
  - $xy + x'z$
  - $A + B'C$
- Each of the $2^n$ functions of $n$ binary variables can be rewritten as a product of maxterms, e.g.:
  $$xy + x'z = (x' + y)(x + z)(y + z)$$
Using Karnaugh Maps to Find Product of Maxterms

\[ F' = xy' + x'z' \]
\[ F = (x'+y)(x+z) \]

Using Karnaugh Maps and Don’t-Care Conditions to Find Product of Maxterms

\[ F' = z' + wy' \]
\[ F = z(w'+y) \]