CSC 370 – Computer Architecture and Organization

Lecture 10: Floating Point Data

Floating Point Representation

- Numbers too large for standard integer representations or that have fractional components are usually represented in scientific notation, a form used commonly by scientists and engineers.
- Examples:

  4.25×10^{1} 10^{-3}

  -3.35×10^{3} -1.42×10^{2}

Normalized Floating Point Numbers

- We are most interested in normalized floating-point numbers, a format which includes:
  - sign
  - significand $(1.0 \leq \text{Significand} < \text{Radix})$
  - integer power of the radix

Examples of Normalized Floating Point Numbers

These are normalized:

- $+1.23456789 \times 10^{1}$
- $-9.987654321 \times 10^{12}$
- $+5.0 \times 10^{0}$

These are ***not*** normalized:

- $+11.3 \times 10^{3}$ *significand > radix*
- -0.0002×10^{7} *significand < 1.0*
- -4.0×10^{½} *exponent not integer*

Converting From Binary To Decimal

$\mathbf{1.00101_2} = 1 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$

$= 1 + 0/2 + 0/4 + 1/8 + 0/16 + 1/32$

$= 1 + 0.125 + 0.03125$

$= \mathbf{1.15625}$

$= 37/32 = 1.15625$

Converting From Decimal To Binary

Let's start with $3.4625 \times 10^1 = 34.625$

Let's deal separately with the 34 (which equals 100010_2)

$2 \times .625 = 1.25$ (save the integer part)

$2 \times .25 = 0.5$ (no integer part to save)

$2 \times .50 = 1.00$ (save the integer part)

Let's write them left to right in order:

$34.625_{10} = 100010.101_2$

Converting From Decimal To Binary – Another Example

$1.23125 \times 10^1 = 12.3125$

$12_{10} = 1100_2$

$2 \times .3125 = \mathbf{0}.625$

$2 \times .625 = \mathbf{1}.25$

$2 \times .25 = \mathbf{0}.50$

$2 \times .50 = \mathbf{1}.0$

$$\mathbf{12.3125_{10} = 1100.0101_2}$$

Normalizing Floating Point Data

Floating point data is normalized so that there is the significand is always one:

$100001.101_2 = 1.00001101 \times 2^5$

$1100.0101_2 = 1.1000101 \times 2^3$

Since the most significant bit is always 1, we can assume that it is implied and that we do not actually have to represent it.

Biased Exponents

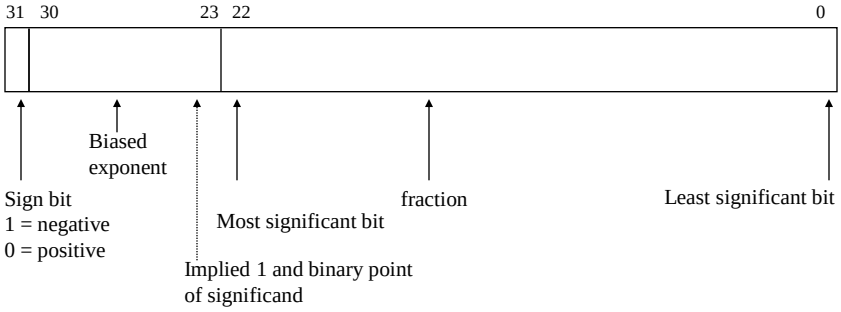
- Short floating point numbers uses 8-bits for the exponents, which we want to range from -128 to +127.
- A biased exponent uses some value other than 0 as the baseline, which must be subtracted to get the actual exponent value.
- Example (in short floating point):
  - exponent 135 = 135 - 127 = 2^8
  - exponent 120 = 120-127 = 2^{-7}

Representing Floating Point Values In Memory

There are three standard formats for representing floating-point numbers:

- 32-bit format (*single-precision*)
- 64-bit format (*double-precision*)
- 80-bit format (*extended precision*)

Short Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

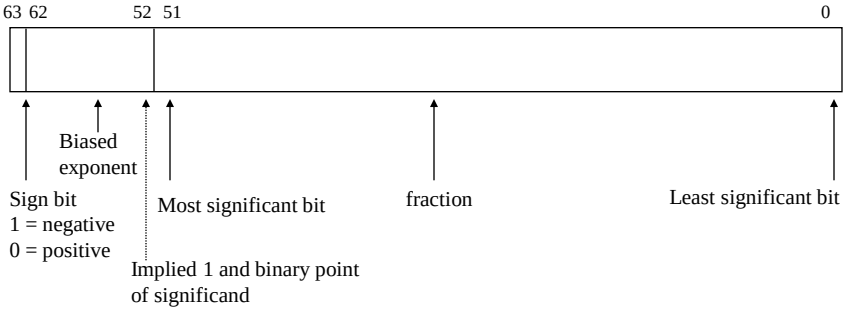
Short: $-1.10001110000\ldots\ 0000_2 \times 2^{3\ +127}$

1 10000010 10001110000… 0000

$1100\ 0001\ 0100\ 0111\ 0000\ \ldots\ 0000_2$

$= C1470000h$

Long Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

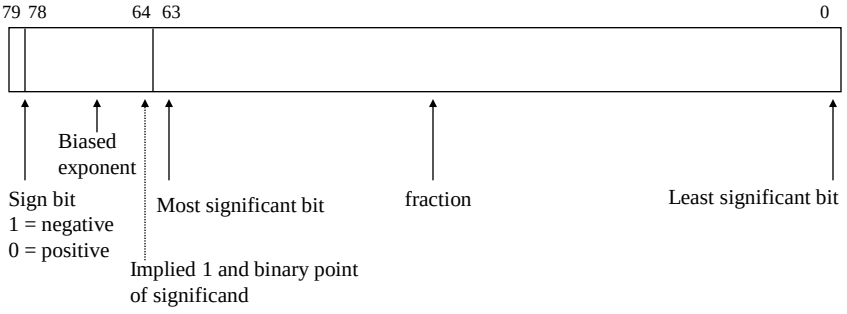
Long: $-1.10001110000\ldots\ 0000_2 \times 2^{3\ +1023}$

1 10000000010 10001110000… 0000

$1100\ 0000\ 0010\ 1000\ 1110\ 0000\ \ldots\ 0000_2$

$= C028E00000000000h$

Extended Floating Point Numbers



Representing Values

$-12.4375_{10} = -1100.0111_2$

Extended: $-1.10001110000\ldots\ 0000_2 \times 2^3$ $^{+16383}$

1 100000000000010 10001110000… 0000

$1100\ 0000\ 0000\ 0010\ 1100\ 0111\ \ 0000\ \ldots\ 0000_2$

$= C002C700000000000000h$

Specifying Floating Point Data In Assembly Language

- We can use the:
  - **dd** (define doubleword) directive to allocate storage for single-precision floats
  - **dq** (define quadword) to allocate storage for double-precision floats and
  - **dt** (define tenbyte) for extended-precision floats.

Specifying Floating Point Data - An Example

- Allocating storage and initializing values

```
ShortOne   dd   1.0
LongOne    dq   1.0
Pi         dd   0.314159265E1
IntRate    dt   13.25E-1
```

Allocating storage without initializing:

```
Mass       dd   ?
CoefFric   dq   ?
Temp       dt   ?
```

Floating Point Operations

- Floating point operations include:
  - moving and rounding data
  - conversion
  - addition
  - subtraction
  - multiplication
  - division
  - remainder
  - comparison

Moving Floating Point Data

- Moving floating point data can be done using the standard **`mov`** instruction in Assembly language.
- If the source and destination are different length, care must be taken in conversion to ensure that exponent and significand are properly converted.

Data Conversion

- Integer and floating point data cannot be used interchangeably; data conversion is necessary and real-to-integer conversion is not without potential problems:
  - Underflow – a magnitude too small to represent as an integer.
  - Overflow – a magnitude too large to represent as an integer.
  - Inexact result – a loss of all of part of the fractional part of the floating-point fvalue.

Floating Point Addition

- To add two floating point values, they have to be aligned so that they have the same exponent.
- After addition, the sum may need to be normalized.
- Potential errors include overflow, underflow and inexact results.
- Examples:

$$\begin{array}{rr} 2.34 \times 10^3 & 6.22 \times 10^8 \\ +\quad 0.88 \times 10^3 & +\ 3.93 \times 10^8 \\ \hline 3.22 \times 10^3 & 10.15 \times 10^8 = 1.015 \times 10^9 \end{array}$$

Floating Point Subtraction

- Subtracting floating point values also requires re-alignment so that they have the same exponent.
- After subtraction, the difference may need to be normalized.
- Potential errors include overflow, underflow and inexact results, and the difference may have one signficant bit less than the operands..
- Examples:

$$\begin{array}{r} 2.34 \times 10^3 \\ \underline{-0.88 \times 10^3} \\ 1.46 \times 10^3 \end{array} \qquad \begin{array}{r} 6.44 \times 10^4 \\ \underline{-\ 6.23 \times 10^4} \\ 0.21 \times 10^4 = 2.1 \times 10^3 \end{array}$$

Floating Point Multiplication

- Multiplying floating point values does not requires re-alignment - realigning may lead to loss of significance.
- After multiplication, the product may need to be normalized.
- Potential errors include overflow, underflow and inexact results.
- Examples:

$$\begin{array}{r} 2.4 \times 10^3 \\ \times\ \underline{6.3 \times 10^{-2}} \\ 15.12 \times 10^1 = 1.512 \times 10^2 \end{array}$$

Floating Point Division

- Dividing floating point values does not requires re-alignment.
- After division, the (floating point) quotient may need to be normalized – there is no remainder
- Potential errors include overflow, underflow, inexact results and attempts to divide by zero.
- Examples:

$$1.86 \times 10^{13} \div 7.44 \times 10^{5} = 0.25 \times 10^{8}$$
$$2.5 \times 10^{7}$$

Floating Point Remainder

- There is usually no remainder in floating point division, because the quotient can be a floating point value itself.
- Sometimes we want a remainder , i.e., the difference between the dividend and the product of the quotient rounded to the nearest integer) and the divisor:
- s REM t = s – t × NINT(s/t)
- Remainder will not produce inexact results, underflow or overflow but can lead to an attempt to divide by zero.

Floating Point Comparison

- There are usually three results that can happen as a result of floating point comparison:
  - less than
  - greater than
  - equal to
- In some instances, there is a fourth result: unordered, which occurs if one of the values is the result of an arithmetic error.
- These errors can result from adding or subtracting infinite values and are called ***NaNs*** (for ***N***ot ***a*** ***N***umber).

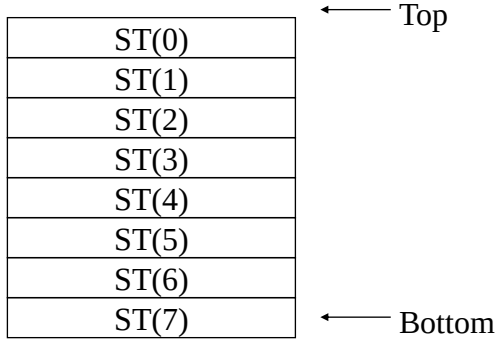
The Intel Floating Point Co-processors

- Early Intel processors (8088/8086, 80286, 80386) had no floating point capabilities; unless you wished to emulate floating point operations using software routines, you needed to add a co-processor (8087, 80287, 80387).
- 80486 and Pentium family processors include a floating point unit with an architecture that is the same as the coprocessors.

The Intel Floating Point Unit

- The Floating Point Unit contains 13 registers:
  - Eight 80-bit data registers (ST(0), ST(1), .., ST(7)) that are usually accessed as a stack with ST(0) representing the top of the stack.
  - Three 16-bit registers, called the tag, control and status registers.
  - Two 32-bit registers that serve as exception pointers.

Data Register



Tag Register

tag 7	tag 6	tag 5	tag 4	tag 3	tag 2	tag 1	tag 0

tag	meaning
00	valid (finite nonzero number)
01	zero
10	invalid (infinite or NaN)
11	empty

Control Register

- The control register contains six exception masks and three control fields
- If one of the exception masks is cleared and that exception occurs, the program is suspended and the an interrupt is generated, which will either correct the problem or terminate the program.
- The control fields control rounding and the type of infinity used.

Floating Point Move Instructions

Floating point ***move*** instructions include:

- **`fld`** *source*– convert to ext. real and push on stack
- **`fild`** *source* – convert from integer and push
- **`fst`** *dest* – store (without popping)
- **`fstp`** *dest* – store and pop
- **`fist`** *dest* – convert to integer and store
- **`fistp`** *dest* – convert to integer, store and pop
- **`fxch`** – exchange ST and ST(1)
- **`fld1`** – push 1 on the stack
- **`fldz`** – push zero on the stack

Floating Point Arithmetic Instructions

- Floating point ***arithmetic*** instructions include:
  - fadd Add
  - fsub Subtract
  - fsubr Subtract Reversed (minuend and subtrahrend are in reverse order)
  - fmul Multiply
  - fdiv Divide
- Without an operand, the operands are popped from the stack and the result is pushed.
- With a single operand, the second operand is specified; the first operand the result are on the top of the stack

Special Floating Point Operations

- In addition to the standard arithmetic operations, there are a few that do not always have integer counterparts:

  `fchs` - change sign (negation)

  `fabs` - absolute value

  `frndint` - round to nearest integer

  `fsqrt` - square root

- In all cases, the operand is on the top of the stack

Using Comparison Instructions

- Comparison instructions set the status bits in the FPU control register. To use these as the basis for a conditional branch, we must load these values into the main flag register.
- We use the sequence:

```
fstsw     StatusReg    ; store status word in
                       ; memory
mov ax, StatusReg      ; Copy into AX register
sahf                   ; copy copy from AH to
                       ; the flag register
```

… … …

```
StatusReg   dw    ?
```

Comparison Instructions

- The comparison instructions set the status bits in the FPU control register depending on the top two values in ST.
- The comparison instructions include:
  - **`fcom`** compares ST with ST(1) or the operand
  - **`fcomp`** compares ST with ST(1) or the operand (and pops ST)
  - **`ficom`** compare ST with an integer operand
  - **`ficomp`** compare ST with an integer operand and pops ST
  - **`ftst`** compares ST to zero

Control Instructions

- Control instructions give the programmer control over the 8087 or FPU.
- They include:
  - **`fwait`** Suspend 8086 (non-FPU) operations until 8087 (or FPU) is not busy
  - **`finit`** Initialize tag, control and status registers.
  - **`fstcw`** Save Control Word (where indicated by operand)

Example: Calculating A Square Root

- Newton's algorithm is frequently used to calculate square root.
- We start with an initial guess that X_0 is the square root of A. We then find a revised guess X1 where:

$$X_1 = (A/X_0 + X_0) / 2$$

- We repeat this until the difference of X_N and X_{N-1} is within an acceptably small.

The Square Root Procedure

```
; procedure to find the square root using Newton's method
; Can be called by programs  in higher languages as well
; Parameters and local values
A             equ     dword ptr [bp + 4]
Xold          equ     dword ptr [bp - 4]
Xnew          equ     dword ptr [bp - 8]
TestResult    equ      word ptr [bp - 10]

.data
Two           dd      2.0
MaxRelErr     dd      0.5E-6


.code
NewSqrt       proc
; set up bp register to point to parameter
              push    bp
              mov     bp, sp
```

```
; allocate stack space for local variables
            sub     sp, 10
; Xnew = 1.0
            fld1
            fstp    Xnew
REPEAT:
; Xold = Xnew
            fld     Xnew
            fst     Xold  ; copy of Xold remains on
                          ; stack
; Xnew = (A/Xold + Xold)
            fld     A
            fld     Xold
            fdiv
            fld     Xold
            fadd
            fld     Two
            fdiv
            fst     Xnew  ; copy of Xnew remains on
                          ; stack




; test MaxRelErr * Xnew > abs(Xnew - Xold)
            fsub
            fabs
            fld     MaxRelErr
            fld     Xnew
            fmul
            fcompp

            fstsw   TestResult
            fwait
            mov     ax, TestResult
            sahf
            jna     REPEAT
; UNTIL MaxRelErr * Xnew > abs(Xnew - Xold)

; return Xnew in FPU stack, restore the non-FPU stack
            fld     Xnew
            add     sp, 10
            pop     bp
            ret     4
newsqrt     endp
            end     newsqrt
```