Data Structures

Lecture 6 - Sorts

What is a sort?

- Sorting is placing a collection of items in order based on a data item (set of data items) called a ***key***.
- Collections are sorted to make it easier to ***search*** for a given item.
- Internal sorting is performed with all the records resident in memory
- External sorting is performed with recorded stored externally.

Efficiency of An Algorithm

- How many times will the innermost loop be performed:

```
for (i = 0;  i < n;  i++)
   for (j = 0;  j < n;  j++)
        for (k = 0;  k < n;  k++)
             cout << x[i][j][k];
```

Answer: n^3 times

O notation

- Given the functions f(n) and g(n), we say that f(n) = *O*(g(n)) if there exists positive integers a and b such that

$$f(n) \leq a \times g(n)\ , \quad \text{for all } n \geq b$$

Examples of *O* notation

- For any constants c, j, k, and l:

  $c = O(1)$

  $c = O(\log n)$ but $\log n \neq O(1)$

  $c \times \log_k n = O(\log n)$

  $c \times \log_k n = O(n)$ but $n \neq O(\log n)$

  $c \times n^k = O(n^k)$ & $c \times n^k = O(n^{k+1})$ but $n^{k+1} \neq O(n^k)$

  $c \times n^j \times (\log_k n)^l = O(n^j (\log n)^l)$

  $c \times n^j \times (\log_k n)^l = O(n^{j+1})$ but $n^{j+1} \neq O(n^j(\log n)^l)$

A Comparison of Some Values

$\log_2 n$	n	$n \log_2 n$	n^2	n^3	2^n
0	1	0	1	1	2
1	2	2	4	8	4
2	4	8	16	64	16
3	8	24	64	512	256
4	16	64	256	4096	65536
5	32	260	1024	32768	4294967296

Bubble Sort

```
/*
 * bubble() - The infamous Bubble Sort
 */
void  bubble(int x[], int n)
{
      int   hold, i, j, pass;
      int   switched = TRUE;

      for  (pass = 0;  pass < n-1 && switched;
                  pass++)                  {
            /* The outer loop controls the
                         number of passes */
```

```
            switched = FALSE;
            /*  The inner loop controls each
                        individual pass */
            for (j = 0;  j < n-pass-1;  j++)
                  if (x[j] > x[j+1])      {
                  /*  Elements are out of order
                              Let's swap  */
                        switched = TRUE;
                        hold = x[j];
                        x[j] = x[j+1];
                        x[j+1] = hold;
                        }
      }
}
```

Tracing the Bubble Sort

Pass	25	57	48	37	12	92	86	33
1	25	48	37	12	57	86	33	92
2	25	37	12	48	57	33	86	92
3	25	12	37	48	33	57	86	92
4	12	25	37	33	48	57	86	92
5	12	25	33	37	48	57	86	92
6	12	25	33	37	48	57	86	92
7	12	25	33	37	48	57	86	92

Efficiency Of The Bubble Sort

- **Best case** - when completely sorted or almost completely sorted = $O(n)$
- **Worst case** - when random or descending order = $(n-1)^2 = n^2 - 2n + 1 = O(n^2)$

Quick Sort

```
/*
 * Quick() - The recursive function which
partitions the array
 *          and sorts the pieces
 */
void  quick(int x[], int lb, int ub)
{
      int   j = 0;

      if (lb >= ub)
            return;
```

```
      partition(x, lb, ub, j);
      quick(x, lb, j-1);
      quick(x, j+1, ub);

}

/*
 * QuickSort() -  The main function for the
 *                quick sort
 */
void  quicksort(int x[], int n)
{
      quick(x, 0, n-1);
}
```

Tracing the Quick Sort

25	57	48	37	12	92	86	33
(12)	25	(57	48	37	92	86	33)
12	25	(48	37	33)	57	(92	86)
12	25	(37	33)	48	57	(92	86)
12	25	(33)	37	48	57	(92	86)
12	25	33	37	48	57	(92	86)
12	25	33	37	48	57	(86)	92
12	25	33	37	48	57	86	92

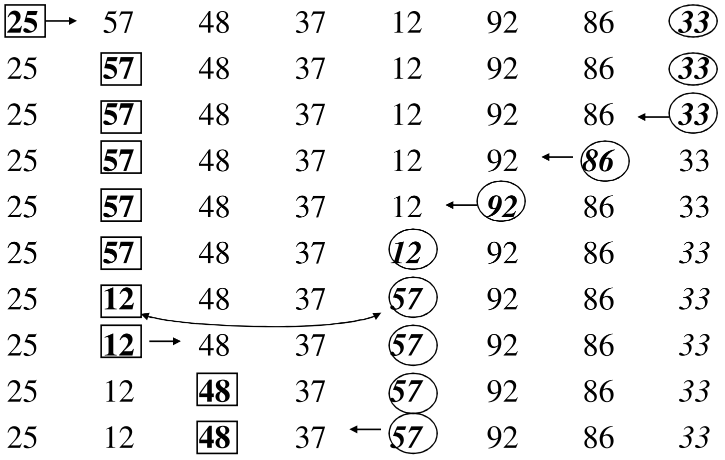
Partition For The Quick Sort

```
/*
 * Partition() - The function used by the Quick
 *                 Sort to partition array around
 *                 the pivot element a
 */
void partition(int x[], int lb, int ub, int &j)
{
      int   a, down, temp, up;
```

```
      /* a is the element whose final
                  partition is sought */
      a = x[lb];

      up = ub;
      down = lb;
      while (down < up) {
            while (x[down] <= a && down < ub)
                  down++;     /* Move up the
                               array */
            while (x[up] > a)
                  --up; /*  Move down the
                              array  */
```

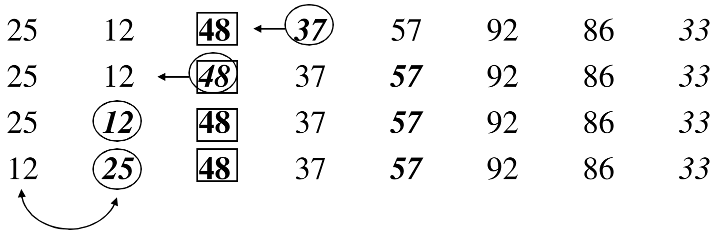
```
            if (down < up)    {
                  /*  Swap x[down] and x[up] */
                  temp = x[down];
                  x[down] = x[up];
                  x[up] = temp;
            }
      }

      x[lb] = x[up];
      x[up] = a;
      j = up;
}
```

Tracing the Partitioning



25	57	48	37	12	92	86	***33***
25	**57**	48	37	12	92	86	***33***
25	**57**	48	37	12	92	86	***33***
25	**57**	48	37	12	92	***86***	33
25	**57**	48	37	12	***92***	86	33
25	**57**	48	37	***12***	92	86	*33*
25	**12**	48	37	***57***	92	86	*33*
25	**12**	48	37	***57***	92	86	*33*
25	12	**48**	37	***57***	92	86	*33*
25	12	**48**	37	***57***	92	86	*33*

Tracing the Partitioning (continued)



25	12	**48**	***37***	57	92	86	*33*
25	12	***48***	37	***57***	92	86	*33*
25	***12***	**48**	37	***57***	92	86	*33*
12	***25***	**48**	37	***57***	92	86	*33*

Efficiency Of The Quick Sort

- **Best case** - when the pivots are all perfectly centered = *O*(n log n)
- **Worst case** - when in perfect order = *O*(n^2)

Selection Sort

```
/*
 * Selection() - The selection sort which puts
 *                the nth largest element in its
 *                proper place
 */
void selection(int x[], int n)
{
      int   i, indx, j, large;
```

```
      for  (i = n-1;  i > 0;  --i)  {
            /*  place the largest number of x[0]
                  through x[i] into large and
                   its index in indx */
            large = x[0];     indx = 0;
            for  (j = 1; j <= i;  j++)
                  if (x[j] > large) {
                        large = x[j];
                        indx = j;
                  }
            x[indx] = x[i];   x[i] = large;
        }
}
```

Tracing the Selection Sort

25	57	48	37	12	92	86	33
25	57	48	37	12	***33***	86	***92***
25	57	48	37	12	33	***86***	92
25	***33***	48	37	12	***57***	86	92
25	***33***	***12***	37	***48***	57	86	92
25	33	12	***37***	48	57	86	92
25	***12***	***33***	37	48	57	86	92
12	***25***	33	37	48	57	86	92

Efficiency Of The Selection Sort

- All cases = $O(n^2)$ - although faster than the Bubble Sort because it makes fewer interchanges.

Heap Sort

```
/*
 * Heapsort() - The heap sort, which first
 *            creates the heap and then uses the
 *           heap to place everything in its
 * proper place.
 */
void  heapsort(int x[], int n)
{
      int   i, elt, son, father, ivalue;
```

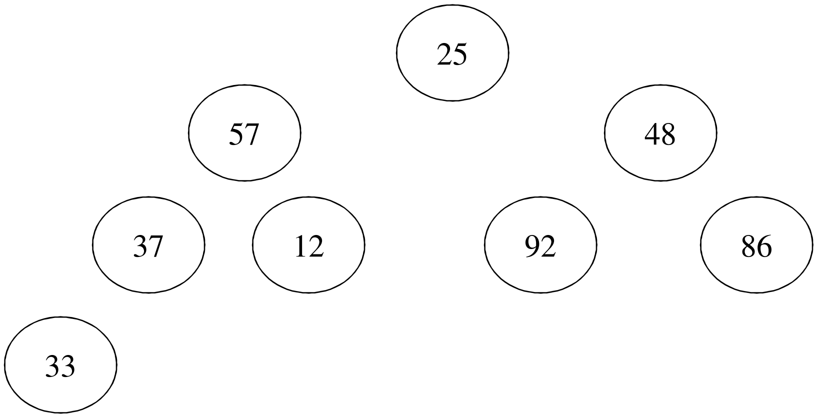
```
/*  Preprocessing phase; create the
             initial heap */
for (i = 1;  i < n;  i++)     {
      elt = x[i];
      /* pqinsert(x, i, elt) */
      son = i;
      father = (son-1)/2;
      while (son > 0 && x[father] < elt){
            x[son] = x[father];
            son = father;
            father = (son-1)/2;
      }
      x[son] = elt;
}
```

```
/*  Selection phase;  Repeatedly
      remove x[0] and insert it in its
      proper position and adjust the
      heap */
for (i = n-1;  i > 0;  --i)    {
      /*  pqmaxdelete(x, i+1) */
      ivalue = x[i];
      x[i] = x[0];
      father = 0;
      /*  son = largeson(0, i-1) */
      if  (i == 1)
            son = -1;
      else
            son = 1;
```

```
            if (i > 2 && x[2] > x[1])
                  son = 2;

            while (son >= 0 && ivalue < x[son]){
                  x[father] = x[son];
                  father = son;

                  /*  son = largeson(f, i-1) */
                  son = 2*father+1;
                  if (son+1 <= i-1 && x[son]
                                    < x[son+1])
                        son++;
```

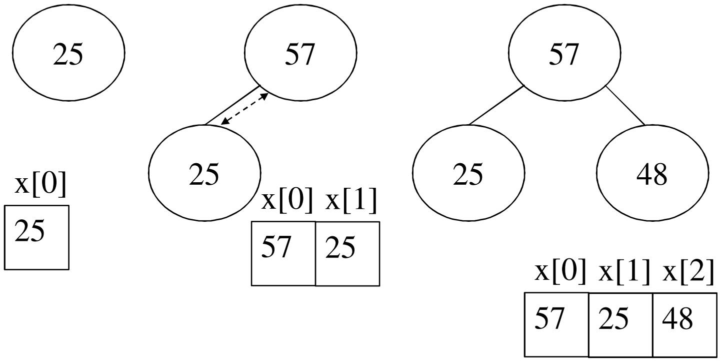
```
                  if (son > i -1)
                              son = -1;
            }
            x[father] = ivalue;
      }
}
```

Original Array



x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
25	57	48	37	12	92	86	33

Building the Heap

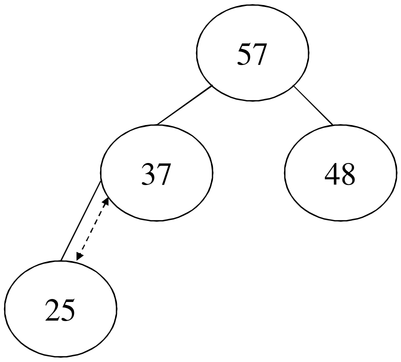


x[0]
25

x[0]	x[1]
57	25

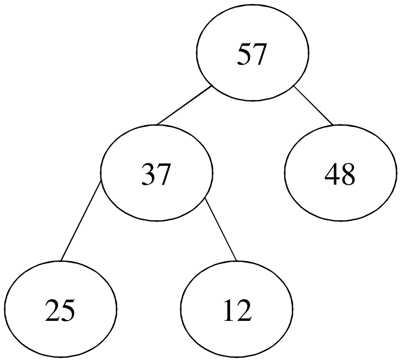
x[0]	x[1]	x[2]
57	25	48

Building the Heap (continued)



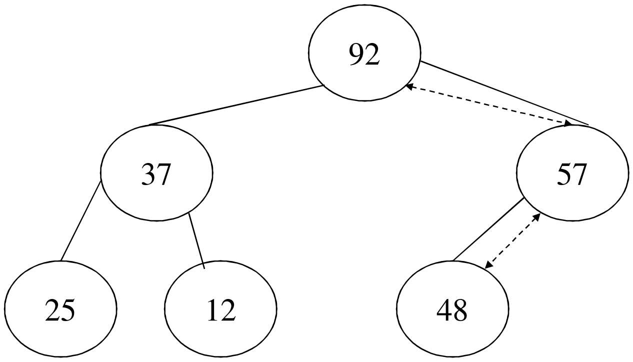
x[0]	x[1]	x[2]	x[3]
57	37	48	25

Building the Heap (continued)



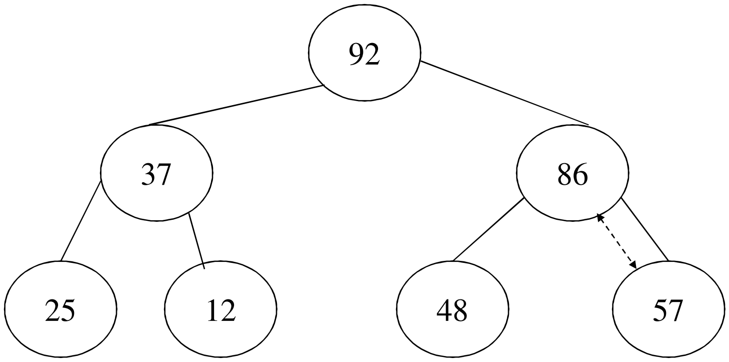
x[0]	x[1]	x[2]	x[3]	x[4]
57	37	48	25	12

Building the Heap (continued)



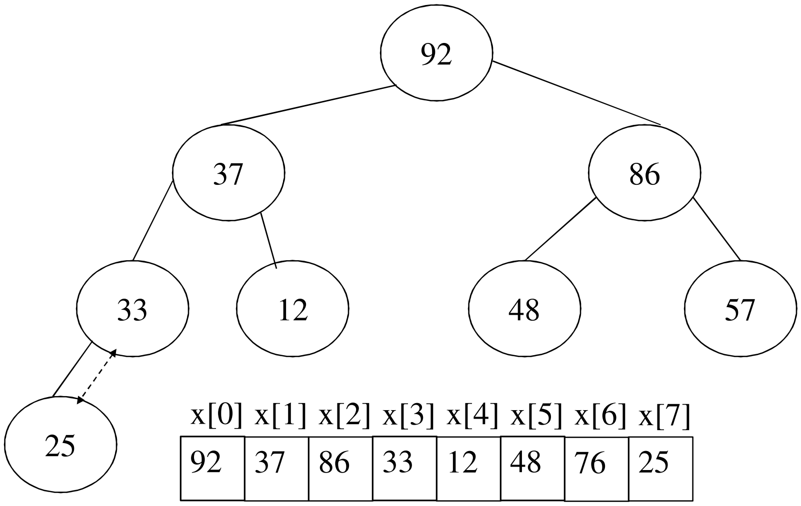
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]
92	37	57	25	12	48

Building the Heap (continued)



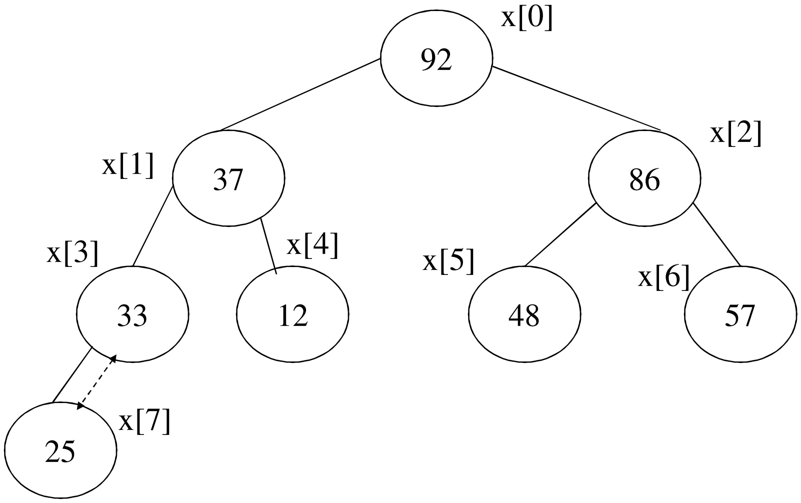
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]
92	37	86	25	12	48	57

Building the Heap (continued)

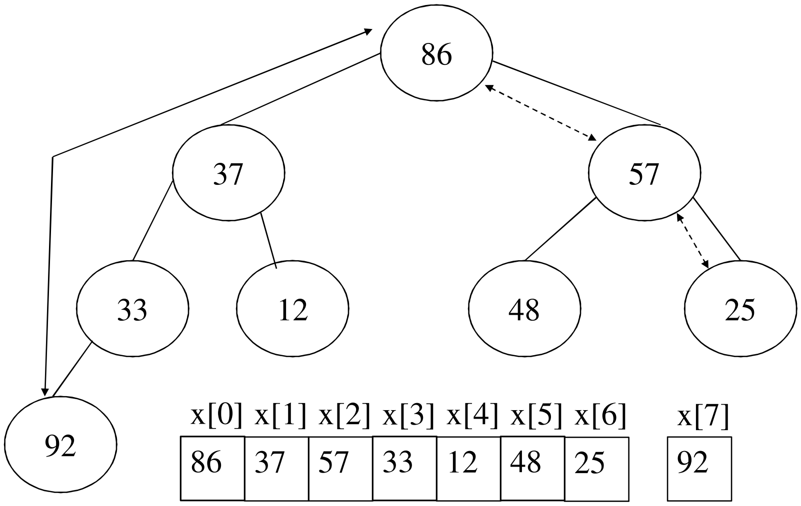


x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
92	37	86	33	12	48	76	25

Using the Heap

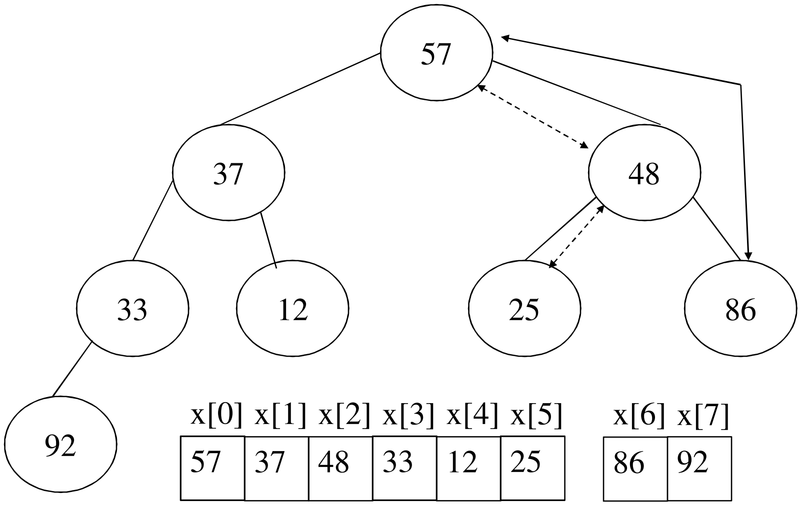


Using the Heap (continued)



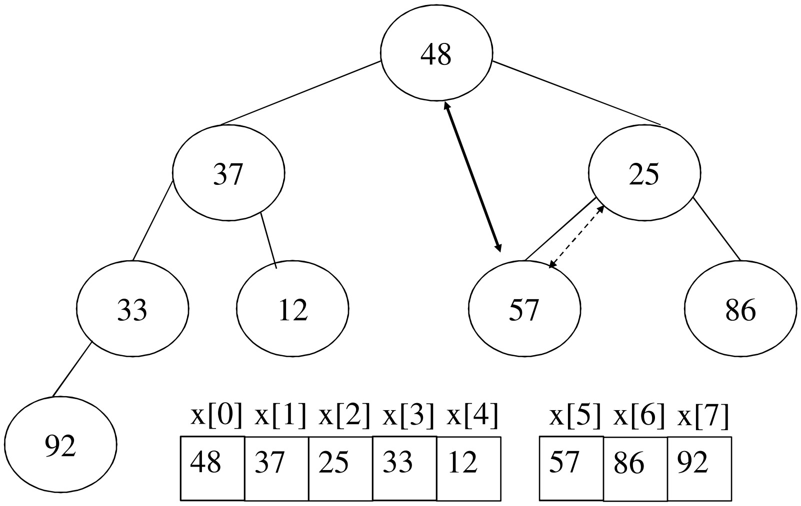
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
86	37	57	33	12	48	25	92

Using the Heap (continued)



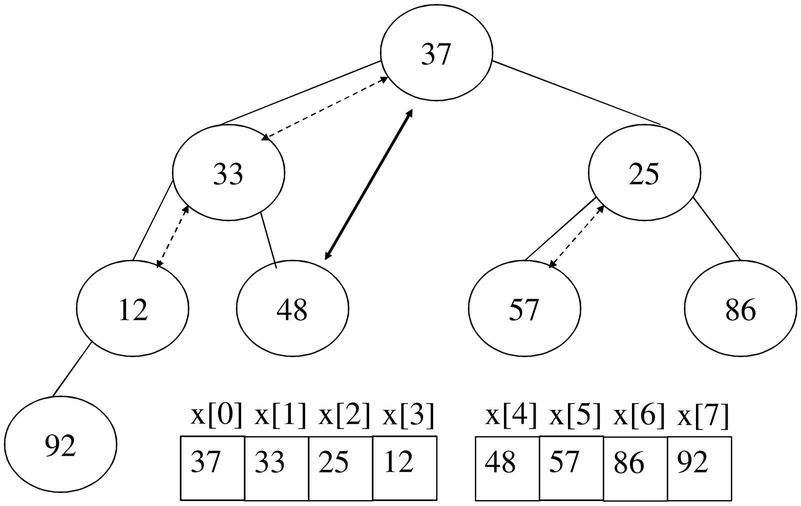
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
57	37	48	33	12	25	86	92

Using the Heap (continued)



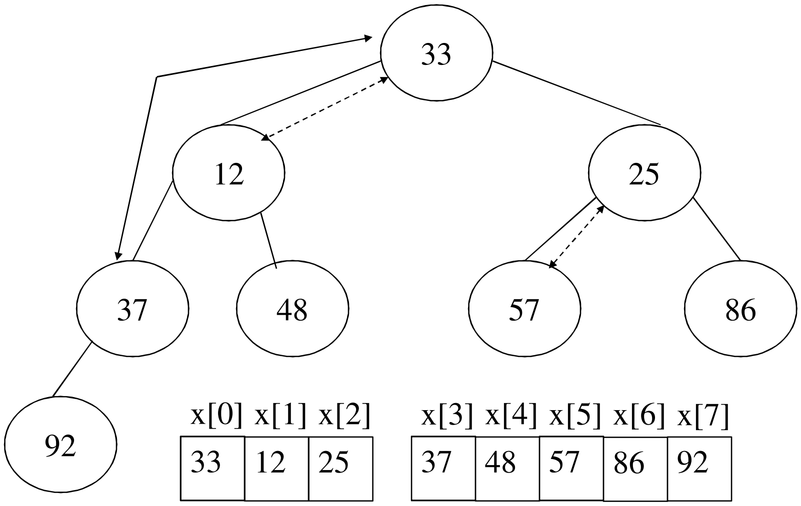
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
48	37	25	33	12	57	86	92

Using the Heap (continued)



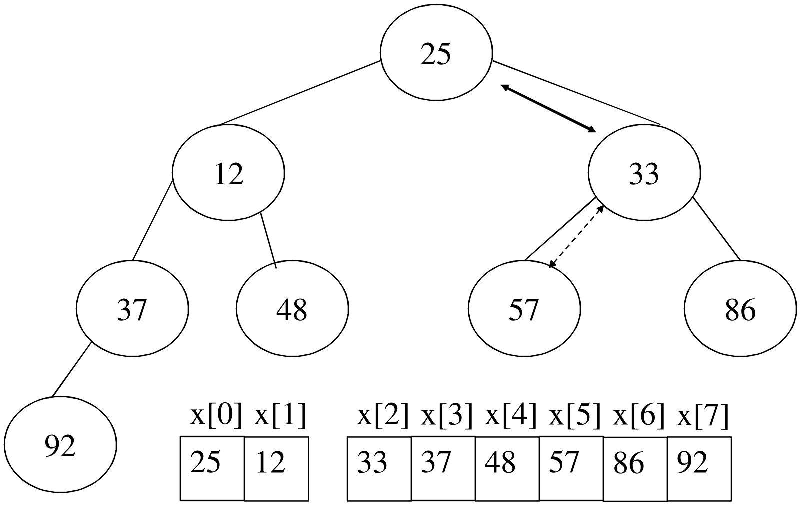
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
37	33	25	12	48	57	86	92

Using the Heap (continued)



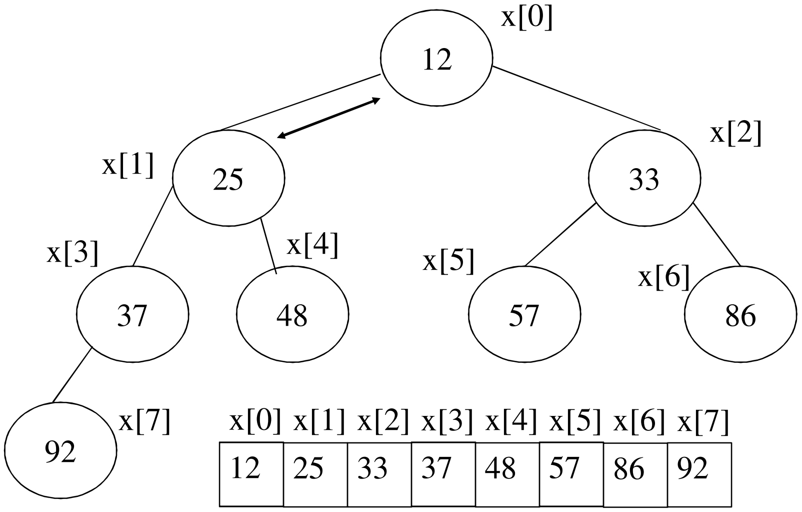
x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
33	12	25	37	48	57	86	92

Using the Heap (continued)



x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
25	12	33	37	48	57	86	92

The Final Heap



x[0]	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]
12	25	33	37	48	57	86	92

Efficiency Of The Heap Sort

- All cases = *O*(n log n) - although slower than the best case of the Quick Sort because of the time it takes to build the Heap and the number of interchanges.

Insertion Sort

```
/*
 * Insertion() - The insertion sort, which
 *          insert a new element one at a time
 *          into a "sorted" file, which
 *          initially has one item.
 */
void  insertion(int x[], int n)
{
      int   i, k, y;
```

```
      /*  x[0] may be thought of as a sorted
            file of one element.  After each
            loop, elements x[0] through x[k]
            are in order */
      for  (k = 1;  k < n;  k++)    {
            /*  Insert x[k] into the file */
            y = x[k];
            /*  Move down 1 position all
                  elements greater than y */
            for  (i = k-1;  i >= 0 && y < x[i];
                                    --i)
                  x[i+1] = x[i];
```

```
            /*  Insert y into its proper
                  position */
                x[i+1] = y;
        }


}
```

Tracing the Insertion Sort

25	57	48	37	12	92	86	33
25	***57***	48	37	12	92	86	33
25	***48***	57	37	12	92	86	33
25	***37***	48	57	12	92	86	33
12	25	37	48	57	92	86	33
12	25	37	48	57	***92***	86	33
12	25	37	48	57	***86***	92	33
12	25	37	48	57	***86***	92	33
12	25	***33***	37	48	57	***86***	92

Efficiency Of The Insertion Sort

- All cases = O(n^2) - although faster than the Bubble Sort because it makes fewer interchanges.

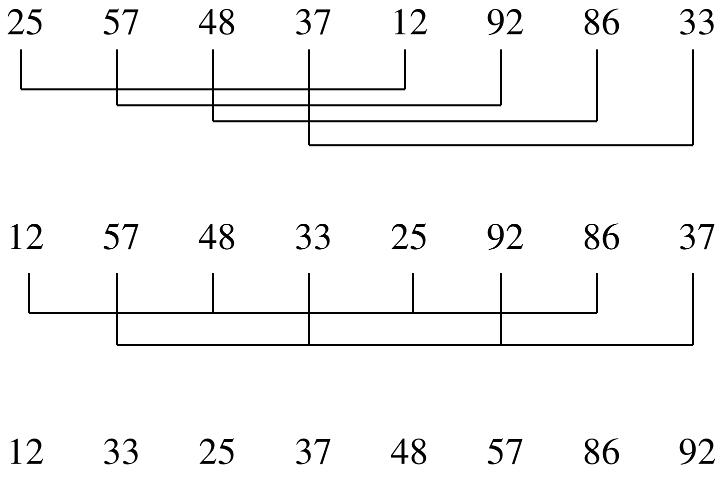
Shell Sort

```
/*
 * ShellSort() - The shell, an insertion sort
 *          which uses a collection of different
 *          increment to merge different data
 *          sets
 */
void  shellsort(int x[], int n)
{
      int   incr, j, k, span, y;
```

```
/* Span is the size of the increment.
      The largest is half the array size
      and it is reduced by half until
      it's 1 */
for (span = n/2;  span >= 1;  span /= 2)
      for (j = span;  j < n;  j++)  {
            /* Insert x[j] into its
                  proper place within
                  its subfile */
            y = x[j];
            for  (k = j-span;
                  k >= 0 && y < x[k];
                  k -= span)
                  x[k+span] = x[k];
```

```
                  x[k+span] = y;
                }
}
```

Tracing the Shell Sort

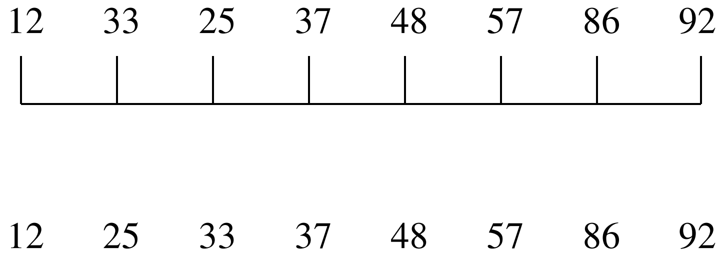


25 57 48 37 12 92 86 33

12 57 48 33 25 92 86 37

12 33 25 37 48 57 86 92

Tracing the Shell Sort (continued)



12 33 25 37 48 57 86 92

12 25 33 37 48 57 86 92

Efficiency Of The Shell Sort

- All cases ≈ $O(n(\log n)^2)$ - although the exact values depends on the set of increments used.

Merge Sort

```
#define           NUMELTS           100
void  mergesort (int x[], int n)
{
      int   aux[NUMELTS], i, j, k,
            l1, l2, size, u1, u2;

      size = 1;   /* Initially merge files of
                   size 1 */
```

```
            while (size < n)  {
                  l1 = 0;     /* Initialize lower
                               bound of 1st file */
                  k = 0;      /* Index for auxiliary
                                    array */
                  /* Check to see if there are two
                        files to merge */
                  while (l1+size < n)     {
                  /* Compute remaining indices */
                        l2 = l1 + size;
                        u1 = l2 - 1;
                        u2 = (l2 + size -1 < n)?
                              l2 + size - 1: n-1;
```

```
                        /* Proceed through the two
                              subfiles */
                        for (i = l1, j = l2;
                        i <= u1 && j <= u2;  k++)
                              if (x[i] <=  x[j])
                                    aux[k] = x[i++];
                              else
                                    aux[k] = x[j++];

                        /*  At this point, one of the
                        two subfiles is exhausted.
                        Insert any remaining portion
                        of the other file */
```

```
                  for  (; i <= u1;  k++)
                        aux[k] = x[i++];
                  for  (; j <= u2;  k++)
                        aux[k] = x[j++];

                  /* Advance l1 to the start of
                  the next pair of subfiles */
                  l1 = u2 + 1;
            }
            /* Copy any remaining single
                  subfile */
            for (i = l1;  k < n;  i++)
                  aux[k++] = x[i];
```

```
            /*  Copy aux back into x and adjust
                  the subfile size */
            for (i = 0;  i < n;  i++)
                  x[i] = aux[i];
                size *= 2;
      }

}
```

Tracing the Merge Sort

[25] [57] [48] [37] [12] [92] [86] [33]

[12 57] [37 48] [25 92] [33 86]

[12 37 48 57] [25 33 86 92]

[12 25 33 37 48 57 86 92]

Efficiency Of The Merge Sort

- The merge sort never requires more than n log n comparisons but requires an extra *O*(n) storage.

Radix Sort

```
/*
 * Power(x, n) - Raises x to the nth power -
 *               not supplied with Turbo C++
 */
int   power(int x, int n)
{
      int   i, product = 1;

      for (i = 0;  i < n;  i++)
            product *= x;
      return(product);
}
```

```
/*
 * RadixSort() - Sorts numbers based by queueing
 *          them repeatedly by a particular
 *          digit (least to most signigficant)
 */
void  radixsort(int x[], int n)
{
      int   front[10], rear[10];
      struct      {
            int   info;
            int   next;
      } node [NUMELTS];
```

```
int   exp, first, i, j, k, p, q, y;

/*  Initialize linked list */
for (i = 0;  i < n-1;  i++)   {
      node[i].info = x[i];
      node[i].next = i+1;
}
node[n-1].info = x[n-1];
node[n-1].next = -1;
first = 0;  /* First is the head of the
                  linked list */
```

```
for  (k = 1;  k < 5;  k++)    {
      /*  Assume that we have four-digit
            numbers */
      /*  Initialize the queues */
      for (i = 0;  i < 10;  i++)    {
            rear[i] = -1;
            front[i] = -1;
      }
```

```
/*  Process each element on the
      list */
while (first != -1)     {
      p = first;
      first = node[first].next;
      y = node[p].info;

      /*  Extract the kth digit */
      exp = power(10, k-1);
      j = (y/exp) % 10;
```

```
      j = (y/exp) % 10;

      /* Insert y into queue[j] */
      q = rear[j];
      if (q == -1)
            front[j] = p;
      else
            node[q].next= p;
            rear[j] = p;
}
```

```
/* At this point, each record is
in its proper queue based on digit k
We now form a single list from all
the queues. */
for  (j = 0;
      j < 10 && front[j] == -1;
      j++)
      ;
first = front[j];
```

```
/*  Link up remaining queues
  First, check to see if finished */
while (j <= 9)    {
      /*  Find the next element */
      for (i = j+1;
            i < 10
                  && front[i] == -1;
            i++)
            ;
      if (i <= 9) {
            p = i;
```

```
                  node[rear[j]].next
                              = front[i];
            }
            j = i;
      }

      node[rear[p]].next = -1;
}
```

```
      /*  Copy back to the original array */
      for (i = 0;  i < n;  i++)     {
            x[i] = node[first].info;
                first = node[first].next;
      }
}
```

Tracing the Radix Sort

25 57 48 37 12 92 86 33

	front	rear
queue[0]		
queue[1]		
queue[2]	12	92
queue[3]	33	
queue[4]		
queue[5]	25	
queue[6]	86	
queue[7]	37	57
queue[8]	48	
queue[9]		

12 92 33 25 86 57 37 48

Tracing the Radix Sort (continued)

12 92 33 25 86 57 37 48

	front	rear
queue[0]		
queue[1]	12	
queue[2]	25	
queue[3]	33	37
queue[4]	48	
queue[5]	57	
queue[6]		
queue[7]		
queue[8]	86	
queue[9]	92	

12 25 33 37 48 57 86 92

Efficiency Of The Radix Sort

- The efficiency = *O*(m * n) storage for m digits and n records, and can approach *O*(n log n)