CSC 273 – Data Structures

Lecture 3- Stacks

What is a stack?

- Some familiar stacks

- Add item on top of stack
- Remove item that is topmost
  - Last In, First Out … LIFO

Specifications of the ADT Stack

Abstract Data Type: Stack		
Data		
• A collection of objects in reverse chronological order and having the same data type		
Operations		
Pseudocode	UML	Description
`push(newEntry)`	`+push(newEntry: T): void`	Task: Adds a new entry to the top of the stack. Input: `newEntry` is the new entry. Output: None.
`pop()`	`+pop(): T`	Task: Removes and returns the stack's top entry. Input: None. Output: Returns the stack's top entry. Throws an exception if the stack is empty before the operation.

Specifications of the ADT Stack

`peek()`	`+peek(): T`	Task: Retrieves the stack's top entry without changing the stack in any way. Input: None. Output: Returns the stack's top entry. Throws an exception if the stack is empty.
`isEmpty()`	`+isEmpty(): boolean`	Task: Detects whether the stack is empty. Input: None. Output: Returns true if the stack is empty.
`clear()`	`+clear(): void`	Task: Removes all entries from the stack. Input: None. Output: None.

Design Decision

- When stack is empty
  - What to do with **`pop`** and **`peek`**?
- Possible actions
  - Assume that the ADT is not empty;
  - Return null.
  - Throw an exception (which type?).

The **`Stack`** Interface

```
// An interface for the ADT stack.

public interface StackInterface<T>
{
   // push() - Adds a new entry to the top of
   //          this stack.
   //          The parameter newEntry  An object
   //          to be added to the stack.
   public void push(T newEntry);
```

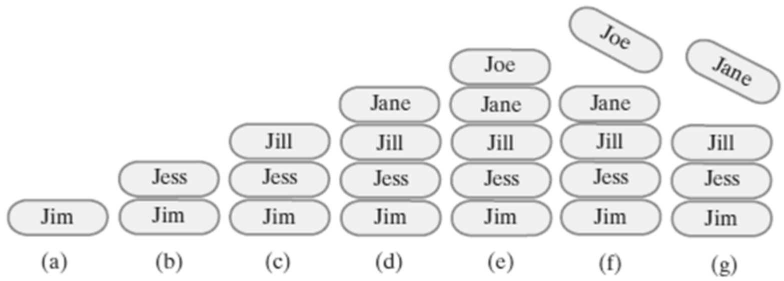
```
   // pop() - Removes and returns this stack's
   //         top entry.
   //         Returns the object at the top of
   //         the stack
   //         Throws  EmptyStackException if the
   //         stack is empty before the
   //         operation.
   public T pop();

   // peek() - Retrieves this stack's top entry.
   //          Returns the object at the top of
   //          the stack.
   //          Throws  EmptyStackException if the
   //          stack is empty. */
   public T peek();
```

```
   // empty() - Detects whether this stack is
   //           empty.
   //            Returns true if the stack is
   // empty
   public boolean isEmpty();

   // clear () - Removes all entries from this
   //            stack
   public void clear();
}
```

Example



(a) push adds Jim; (b) push adds Jess;
(c) push adds Jill; (d) push adds Jane;
(e) push adds Joe; (f) pop retrieves and removes Joe;
(g) pop retrieves and removes Jane

Security Note

- Design guidelines
  - Use preconditions and postconditions to document assumptions.
  - Do not trust client to use public methods correctly.
  - Avoid ambiguous return values.
  - Prefer throwing exceptions instead of returning values to signal problem.

Notations for Algebraic Expressions

- There are 3 different ways to write an algebraic expressions:
  - Infix – Standard algebra – 2 operands with the operator in the middle
  - Prefix – operator followed by 2 operands - aka ***Polish notation***
  - Postfix – 2 operands followed by an operator – aka ***Reverse Polish notation***

Examples of Infix, Prefix, Postfix

- Infix A + B, 3*x – y
- Prefix +AB, -*3xy
- Postfix AB+, 3x*y-

Converting to Infix to Postfix

A + B – C => A + [B*C]

=> A+[BC*]

=> A [BC*] +

=> ABC * +

Converting to Infix to Prefix

A + B * C => A + [B*C]

=> A+[*BC]

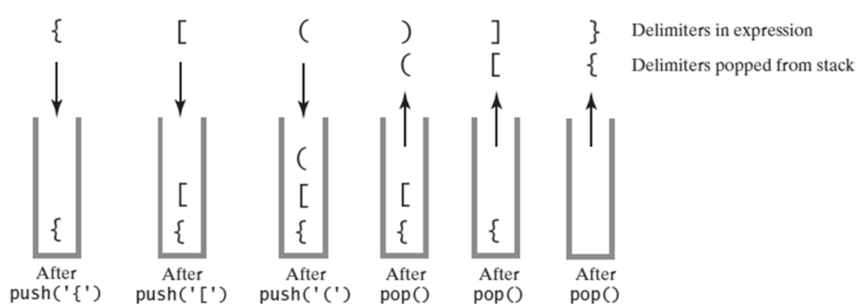
=> + A [*BC]

=> + A * BC

Conversion Examples

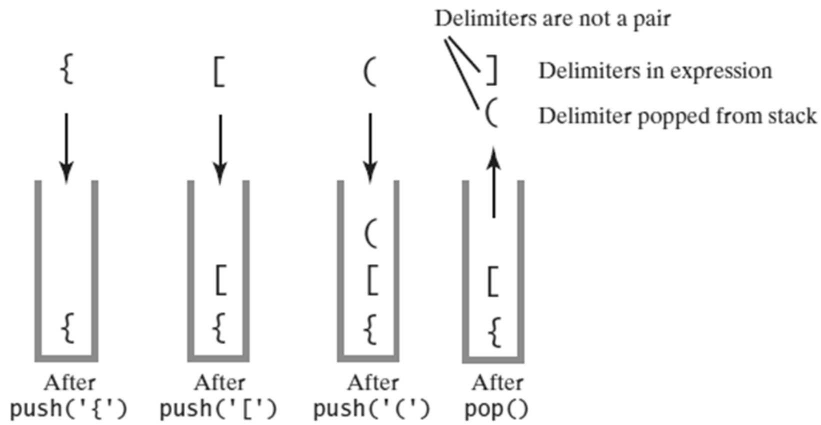
Infix	Prefix	Postfix
A + B	+ A B	A B +
A + B – C	- + A B C	A B + C –
(A+B)*(C-D)	* + A B – C D	A B + C D - *
A^B*C-D+E/F/(G+H)	+-*^ABCD//EF+GH	AB^C*D-EF/GH+/

Processing Algebraic Expressions



Scanning an expression that contains the balanced delimiters
{ [( )] }

Processing Algebraic Expressions



Scanning an expression that contains the balanced delimiters
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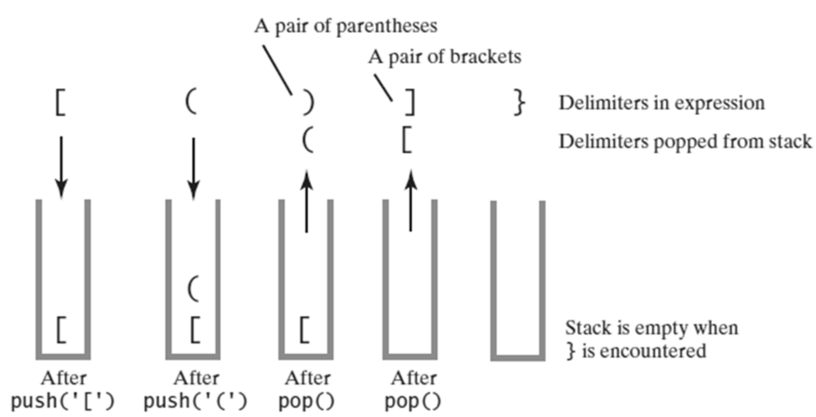
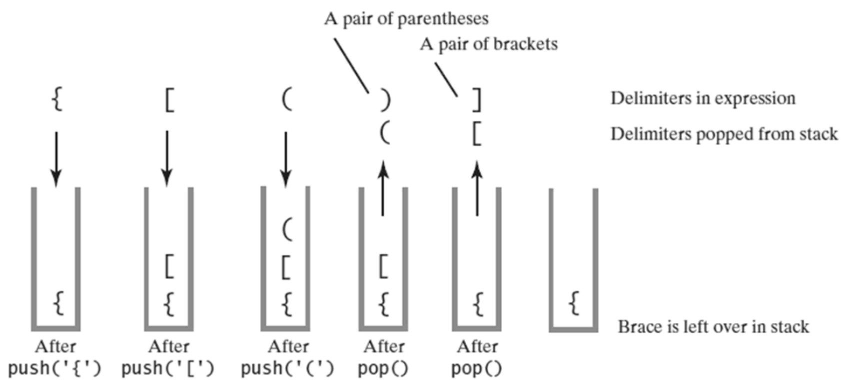
Processing Algebraic Expressions



Scanning an expression that contains the balanced delimiters
[( )] }

Processing Algebraic Expressions



Scanning an expression that contains the balanced delimiters
{ [( )]

Algorithm for Processing Algebraic Expressions

```
Algorithm checkBalance(expression)
// Returns true if the parentheses, brackets, and braces in an expression are paired correctly.

isBalanced = true
while ((isBalanced == true) and not at end of expression)
{
   nextCharacter = next character in expression
   switch (nextCharacter)
   {
      case '(': case '[': case '{':
         Push nextCharacter onto stack
         break

      case ')': case ']': case '}':
         if (stack is empty)
            isBalanced = false
         else
```

Algorithm for Processing Algebraic Expressions

```
      case ')': case ']': case '}':
         if (stack is empty)
            isBalanced = false
         else
         {
            openDelimiter = top entry of stack
            Pop stack
            isBalanced = true or false according to whether openDelimiter and
                             nextCharacter are a pair of delimiters
         }
         break
   }
}

if (stack is not empty)
   isBalanced = false
return isBalanced
```

BalanceChecker()

```
// A class that checks whether the parentheses,
// brackets, and braces in a string occur in
// left/right pairs.

public class BalanceChecker  {
  // Decides whether the parentheses, brackets,
  // and braces in a string occur in left/right
  // pairs.
  // Returns true if the delimiters are paired
  // correctly
  public static boolean
        checkBalance(String expression)    {
    StacInterface<Character>
       openDelimiterStack = new ArrayStack<>();
```

```
int characterCount = expression.length();
boolean isBalanced = true;
int index = 0;
char nextCharacter = ' ';

while (isBalanced &&
        (index < characterCount))     {
  nextCharacter = expression.charAt(index);
  switch (nextCharacter)       {
  case '(': case '[': case '{':
    openDelimiterStack.push(nextCharacter);
    break;
```

```
  case ')': case ']': case '}':
    if (openDelimiterStack.isEmpty())
      isBalanced = false;
    else  {
      char openDelimiter
            = openDelimiterStack.pop();
      isBalanced = isPaired(openDelimiter,
                            nextCharacter);
    }
  break;

  // Ignore unexpected characters
  default: break;
  }
  index++;
}
```

```java
    if (!openDelimiterStack.isEmpty())
      isBalanced = false;

    return isBalanced;
  }
```

```java
// Returns true if the given characters, open and
// close, form a pair of parentheses, brackets,
// or braces.
private static boolean isPaired(char open,
                                  char close)   {
      return (open == '(' && close == ')') ||
             (open == '[' && close == ']') ||
             (open == '{' && close == '}');
   }
}
```

Converting `a + b * c` Postfix

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
+	*a*	+
b	*a b*	+
*	*a b*	+ *
c	*a b c*	+ *
	a b c *	+
	a b c * +	

Converting `a - b + c` Postfix

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
−	*a*	−
b	*a b*	−
+	*a b* −	
	a b −	+
c	*a b* − *c*	+
	a b − *c* +	

Converting a ^ b ^ c Postfix

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
^	*a*	^
b	*a b*	^
^	*a b*	^^
c	*a b c*	^^
	a b c ^	^
	a b c ^ ^	

Infix to Postfix Conversion

• Operand	Append each operand to the end of the output expression.
• Operator ^	Push ^ onto the stack.
• Operator +, -, *, or /	Pop operators from the stack, appending them to the output expression, until the stack is empty or its top entry has a lower precedence than the new operator. Then push the new operator onto the stack.
• Open parenthesis	Push (onto the stack.
• Close parenthesis	Pop operators from the stack and append them to the output expression until an open parenthesis is popped. Discard both parentheses.

Infix to Postfix Conversion Algorithm

```
Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression.

operatorStack = a new empty stack
postfix = a new empty string
while (infix has characters left to parse)
{
   nextCharacter = next nonblank character of infix
   switch (nextCharacter)
   {
      case variable:
         Append nextCharacter to postfix
         break

      case '^' :
         operatorStack.push(nextCharacter)
         break
```

Infix to Postfix Conversion Algorithm

```
case '+' : case '-' : case '*' : case '/' :
   while (!operatorStack.isEmpty() and
          precedence of nextCharacter <= precedence of operatorStack.peek())
   {
      Append operatorStack.peek() to postfix
      operatorStack.pop()
   }
   operatorStack.push(nextCharacter)
   break

case '( ' :
   operatorStack.push(nextCharacter)
   break

case ')' : // Stack is not empty if infix expression is valid
   topOperator = operatorStack.pop()
   while (topOperator != '(')
   {
```

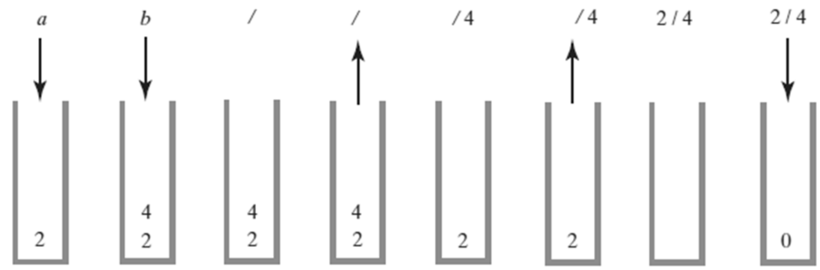
Infix to Postfix Conversion Algorithm

```
            Append topOperator to postfix
            topOperator = operatorStack.pop()
         }
         break
      default: break // Ignore unexpected characters
   }
}
while (!operatorStack.isEmpty())
{
   topOperator = operatorStack.pop()
   Append topOperator to postfix
}
return postfix
```

Converting **a / b * (c + (d – e))** to Postfix

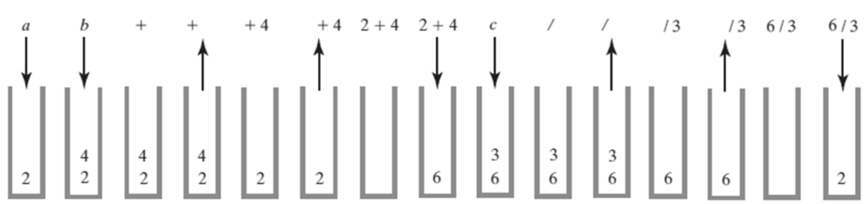
Next Character from Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
/	*a*	/
b	*a b*	/
*	*a b* /	
	a b /	*
(	*a b* /	* (
c	*a b* / *c*	* (
+	*a b* / *c*	* (+
(	*a b* / *c*	* (+ (
d	*a b* / *c d*	* (+ (
[illegible]	[illegible]	*a b* / *c d e*
[illegible]	[illegible]	*a b* / *c d e* –
[illegible]	[illegible]	*a b* / *c d e* –
[illegible]	[illegible]	*a b* / *c d e* – –
[illegible]	[illegible]	*a b* / *c d e* – –
[illegible]	[illegible]	*a b* / *c d e* – –

Evaluating Postfix Expressions



Evaluating **`a b /`** where **`a`** = 4 and **`b`** = 2

Evaluating Postfix Expressions



Evaluating **`a b + c /`** when *a* is 2, *b* is 4, and *c* is 3

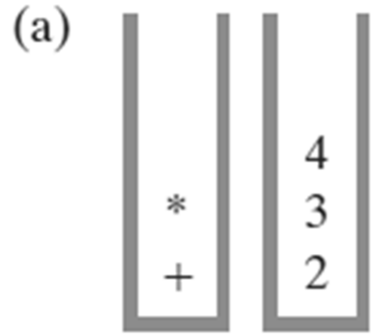
Algorithm for Evaluating Postfix Expressions

```
Algorithm evaluatePostfix(postfix)
// Evaluates a postfix expression.

valueStack = a new empty stack
while (postfix has characters left to parse)
{
    nextCharacter = next nonblank character of postfix
    switch (nextCharacter)
    {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break
      case '+' : case '-' : case '*' : case '/' : case '^' :
```

Algorithm for Evaluating Postfix Expressions

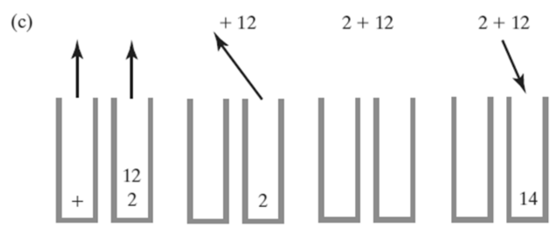
```
         break
      case '+' : case '-' : case '*' : case '/' : case '^' :
         operandTwo = valueStack.pop()
         operandOne = valueStack.pop()
         result = the result of the operation in nextCharacter and its operands
                  operandOne and operandTwo
         valueStack.push(result)
         break
      default: break // Ignore unexpected characters
    }
}
```

Evaluating **a + b * c** when **a** is 2, **b** is 3, and **c** is 4



Evaluating **a + b * c** – Performing the Multiplication

Evaluating `a + b * c` –
Performing the Addition



Algorithm for Evaluating infix Expressions

```
Algorithm evaluateInfix(infix)
// Evaluates an infix expression.

operatorStack = a new empty stack
valueStack = a new empty stack
while (infix has characters left to process)
{
   nextCharacter = next nonblank character of infix
   switch (nextCharacter)
   {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break
      case '^' :
         operatorStack.push(nextCharacter)
         break
      case '+' : case '-' : case '*' : case '/' :
         while (!operatorStack.isEmpty() and
```

Algorithm for Evaluating infix Expressions

```
case '+' : case '-' : case '*' : case '/' :
   while (!operatorStack.isEmpty() and
         precedence of nextCharacter <= precedence of operatorStack.peek())
   {
      // Execute operator at top of operatorStack
      topOperator = operatorStack.pop()
      operandTwo = valueStack.pop()
      operandOne = valueStack.pop()
      result = the result of the operation in topOperator and its operands
               operandOne and operandTwo
      valueStack.push(result)
   }
   operatorStack.push(nextCharacter)
   break

case '(' :
   operatorStack.push(nextCharacter)
   break

case ')' : // Stack is not empty if infix expression is valid
```

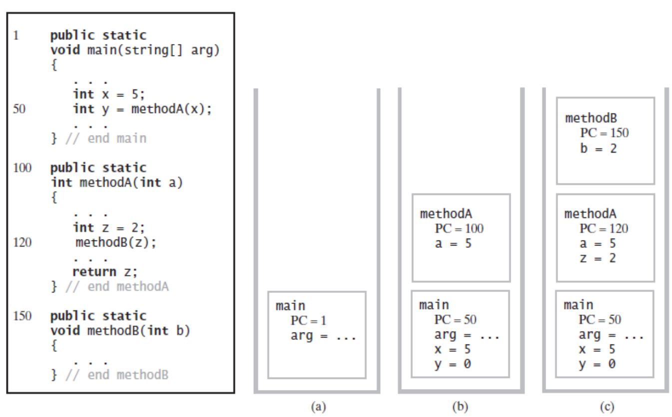
Algorithm for Evaluating infix Expressions

```
case '(' :
   operatorStack.push(nextCharacter)
   break

case ')' : // Stack is not empty if infix expression is valid
   topOperator = operatorStack.pop()
   while (topOperator != '(')
   {
      operandTwo = valueStack.pop()
      operandOne = valueStack.pop()
      result = the result of the operation in topOperator and its operands
               operandOne and operandTwo
      valueStack.push(result)
      topOperator = operatorStack.pop()
   }
   break
```

Algorithm for Evaluating infix Expressions

```
        default: break // Ignore unexpected characters
    }
}
while (!operatorStack.isEmpty())
{
  topOperator = operatorStack.pop()
  operandTwo = valueStack.pop()
  operandOne = valueStack.pop()
  result = the result of the operation in topOperator and its operands
           operandOne and operandTwo
  valueStack.push(result)
}
return valueStack.peek()
```

The Program Stack



Program

Program stack at three points in time (PC is the program counter)

(a) when main begins execution;
(b) when methodA begins execution;
(c) when methodB begins execution

Java Class Library: The Class `Stack`

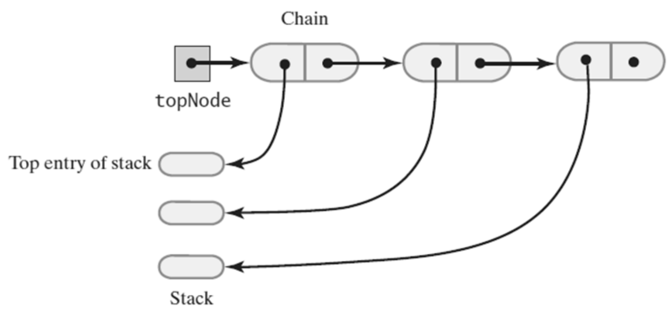
- Found in **`java.util`**
- Methods
  - A constructor – creates an empty stack
  - **`public T push(T item);`**
  - **`public T pop();`**
  - **`public T peek();`**
  - **`public boolean empty();`**

Linked Implementation

- Each operation involves top of stack
  - **`push`**
  - **`pop`**
  - **`peek`**
- Head of linked list easiest, fastest to access
  - Let this be the top of the stack

Linked Implementation

A chain of linked nodes that implements a stack



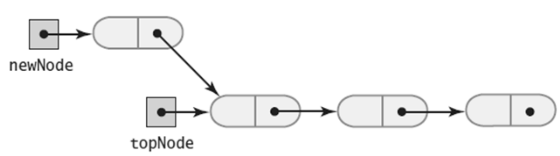
Linked Implementation

```
// A class of stacks whose entries are stored in
// a chain of nodes.

public final class LinkedStack<T>
                  implements StackInterface<T> {
  // References the first node in the chain
  private Node topNode;

  public LinkedStack()  {
    topNode = null;
  }
  // Stack operations go here
```

```
  private class Node    {
    private T    data; // Entry in stack
    private Node next; // Link to next node

    // Constructors, accessors and mutators go
    // here
  }
}
```

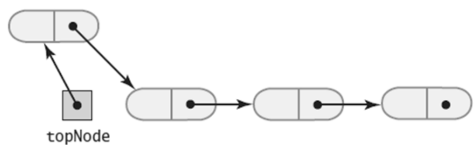
Linked Implementation

A new node that references the node at the top of the stack



Linked Implementation

The new node is now at the top of the stack

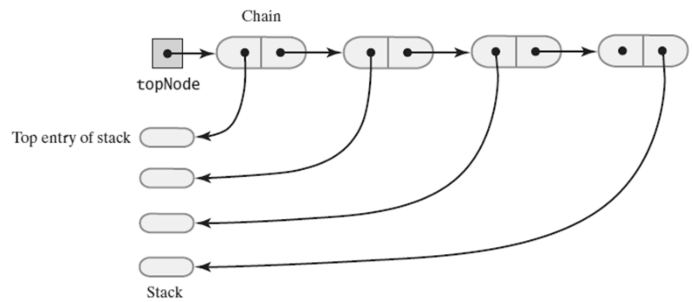


push()

```
public void push(T newEntry)      {
  Node newNode = new Node(newEntry, topNode);
  topNode = newNode;
}
```

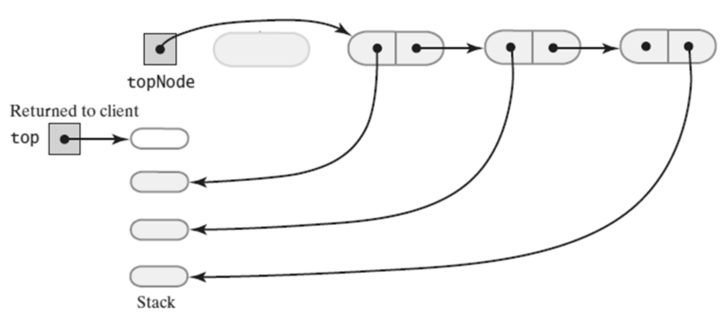
Linked Implementation

The stack before the first node in the chain is deleted



Linked Implementation

The stack after the first node in the chain is deleted



pop()

```
public T pop()  {
  // Might throw EmptyStackException
  T top = peek();

  assert (topNode != null);
  topNode = topNode.getNextNode();
  return top;
}
```

isEmpty() and clear()

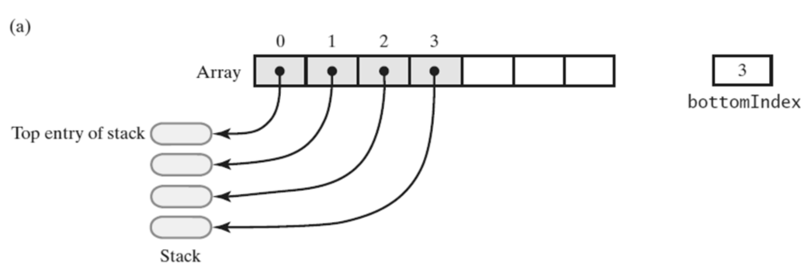
```
public boolean isEmpty()    {
  return topNode == null;
}

public void clear()   {
  // Causes deallocation of nodes in the chain
  topNode = null;
}
```

Array-Based Implementation

- Each operation involves top of stack
  - **`push`**
  - **`pop`**
  - **`peek`**
- End of the array easiest to access
  - Let this be top of stack
  - Let first entry be bottom of stack

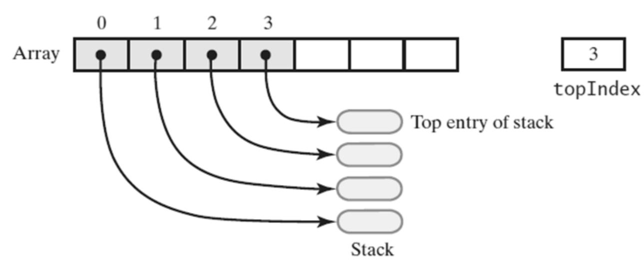
Array Implementation

An array that implements a stack; its first location references the top entry in the stack;



Array Implementation

An array that implements a stack; its first location references the bottom entry in the stack



Array Implementation

```
import java.util.Arrays;
import java.util.EmptyStackException;
// A class of stacks whose entries are stored in
// an array.

public final class ArrayStack<T>
           implements StackInterface<T>  {
  // Array of stack entries
  private T[] stack;

  // Index of top entry
  private int topIndex;
  private boolean initialized = false;
```

```
private static final int DEFAULT_CAPACITY = 50;
private static final int MAX_CAPACITY = 10000;

public ArrayStack()   {
  this(DEFAULT_CAPACITY);
}
```

```
public ArrayStack(int initialCapacity)  {
  checkCapacity(initialCapacity);

  // The cast is safe because the new array
  // contains null entries
  @SuppressWarnings("unchecked")
  T[] tempStack
        = (T[]) new Object[initialCapacity];

  stack = tempStack;
  topIndex = -1;
  initialized = true;
}

// Implementations for stack operations and
// checkCapacity and checkInitialization go
// here
```

Adding to the top - **push()**

```
public void push(T newEntry)      {
  checkInitialization();
  ensureCapacity();
  stack[topIndex + 1] = newEntry;
  topIndex++;
}
```

Adding to the top - **checkCapacity()**

```
private void checkCapacity(int capacity)    {
  if (capacity > MAX_CAPACITY)
    throw new IllegalStateException
      ("Attempt to create a stack "
          + "whose capacity exceeds "
          + "allowed maximum.");
}
```

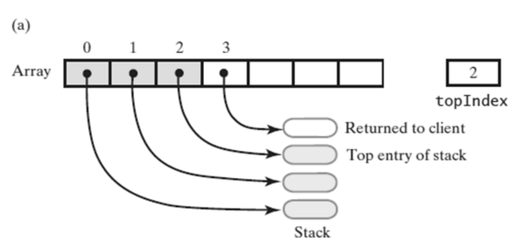
Array-Based Implementation

Retrieving the top, the operation is O(1)

```
public T peek()
{
   checkInitialization();
   if (isEmpty())
      throw new EmptyStackException();
   else
      return stack[topIndex];
} // end peek
```

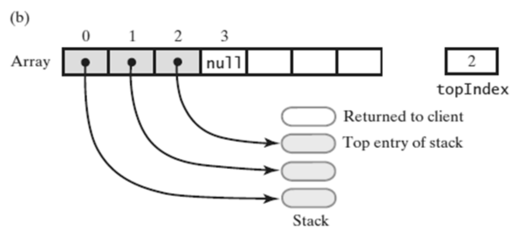
Array-Based Implementation

An array-based stack after its top entry is removed by decrementing **`topIndex`**;



Array-Based Implementation

An array-based stack after its top entry is removed by setting **`stack[topIndex]`** *to null and then decrementing* **`topIndex`**



Array-Based Implementation

Removing the top

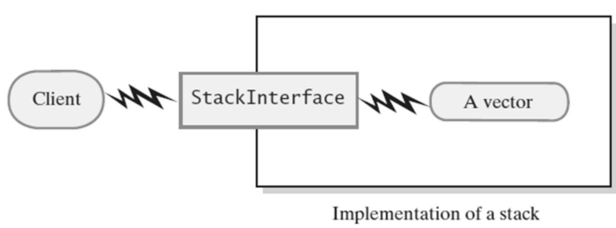
```
public T pop()
{
  checkInitialization();
  if (isEmpty())
     throw new EmptyStackException();
  else
  {
     T top = stack[topIndex];
     stack[topIndex] = null;
     topIndex--;
     return top;
  } // end if
} // end pop
```

Vector Based Implementation

- Vector: an object that behaves like a high-level array
  - Index begins with 0
  - Methods to access or set entries
  - Size will grow as needed
- Use vector's methods to manipulate stack

Vector Based Implementation

A client using the methods given in **`StackInterface`**; *these methods interact with a vector's methods to perform stack operations*



The Class **`Vector`**

- Constructors
- Has methods to add, remove, clear
- Also methods to determine
  - Last element
  - Is the vector empty
  - Number of entries

Vector Implementation

```
import java.util.Vector;
import java.util.EmptyStackException;

//  A class of stacks whose entries are stored
//  in a vector.
public final class VectorStack<T>
         implements StackInterface<T>  {
  // Last element is the top entry in stack
  private Vector<T> stack;
  private boolean initialized = false;
  private static final int DEFAULT_CAPACITY = 50;
  private static final int MAX_CAPACITY = 10000;
```

```
  public VectorStack()  {
    this(DEFAULT_CAPACITY);
  }

  public VectorStack(int initialCapacity)     {
    checkCapacity(initialCapacity);

     // Size doubles as needed
    stack = new Vector<>(initialCapacity);
    initialized = true;
  }

  // Implementations of stack operations and
  // checkInitialization and checkCapacity go
  // here
}
```

Vector Implementation

Adding to the top

```
  public void push(T newEntry)     {
    checkInitialization();
    stack.add(newEntry);
  }
```

peek()

Retrieving the top

```
public T peek()    {
  checkInitialization();
  if (isEmpty())
    throw new EmptyStackException();
  else
    return stack.lastElement();
}
```

pop()

Removing the top

```
public T pop()   {
  checkInitialization();
  if (isEmpty())
    throw new EmptyStackException();
  else
    return stack.remove(stack.size() - 1);
}
```

Rest of the VectorStack Class

```
public boolean isEmpty()     {
  return stack.isEmpty();
}

public void clear()     {
  stack.clear();
}
```