CSC 273 – Data Structures

Lecture 2- Efficiency of Algorithms

Why Efficient Code?

- Computers are faster, have larger memories
  - So why worry about efficient code?
- And … how do we measure efficiency?

Example – Sum of First *n* Values

- Consider the problem of summing

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ ... + n$$

How would we code this?

Example – Sum of First *n* Values

<u>Approach A</u>

```
sum = 0;
for (i = 1;  i <= n;  i++)
      sum = sum = i;
```

<u>Approach B</u>

```
sum = 0;
for (i = 1;  i <= n;  i++)
      for (j = 1;  j <= i; j++)
            sum = sum = 1;
```

<u>Approach C</u>

```
Sum = n * (n+1) / 2
```

Sum of First *n* Numbers

```
public static void main(String[] args) {
       long n  = 10000;
       long sum = 0;

       // Algorithm A
       for (long i = 1;  i<= n; i++)
           sum = sum + i;
       System.out.println("Sum is " + sum);
```

```
       sum = 0;
       // Algorithm B
       for (long i = 1;  i <= n;  i++)
           for (long j = 1; j <= i; j++)
               sum = sum + 1;
       System.out.println("Sum is " + sum);

       sum = 0;
       // Algorithm C
       sum = n * (n + 1) / 2;
       System.out.println("Sum is " + sum);
   }
```

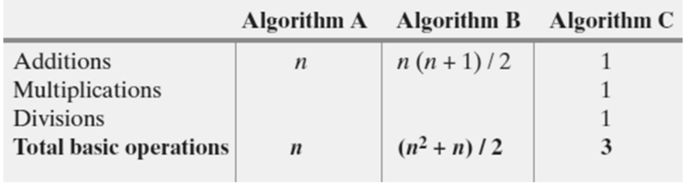
What is “best”?

- An algorithm has both time and space constraints – that is complexity
  - Time complexity
  - Space complexity
- This study is called analysis of algorithms

Counting Basic Operations

- A basic operation of an algorithm
  - The most significant contributor to its total time requirement

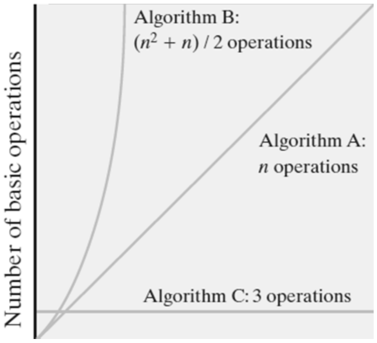
Number of required basic operations



	Algorithm A	Algorithm B	Algorithm C
Additions	n	$n(n+1)/2$	1
Multiplications			1
Divisions			1
Total basic operations	n	$(n^2+n)/2$	**3**

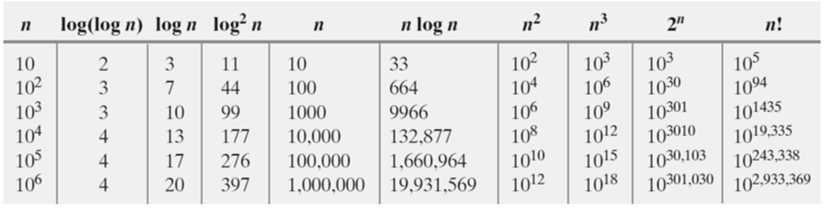
Counting Basic Operations

Number of basic operations required by the algorithm



Counting Basic Operations

Typical growth-rate functions evaluated at increasing values of *n*



n	$\log(\log n)$	$\log n$	$\log^2 n$	n	$n \log n$	n^2	n^3	2^n	$n!$
10	2	3	11	10	33	10^2	10^3	10^3	10^5
10^2	3	7	44	100	664	10^4	10^6	10^{30}	10^{94}
10^3	3	10	99	1000	9966	10^6	10^9	10^{301}	10^{1435}
10^4	4	13	177	10,000	132,877	10^8	10^{12}	10^{3010}	$10^{19,335}$
10^5	4	17	276	100,000	1,660,964	10^{10}	10^{15}	$10^{30,103}$	$10^{243,338}$
10^6	4	20	397	1,000,000	19,931,569	10^{12}	10^{18}	$10^{301,030}$	$10^{2,933,369}$

Best, Worst, and Average Cases

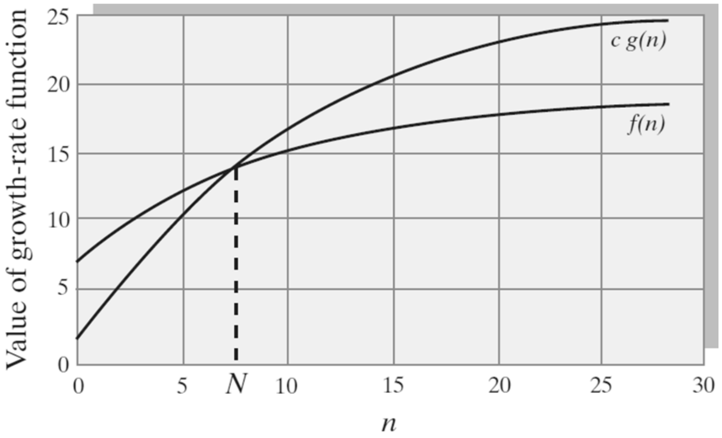
- For some algorithms, execution time depends only on size of data set
- Other algorithms depend on the nature of the data itself
  - Here we seek to know best case, worst case, average case

Big Oh Notation

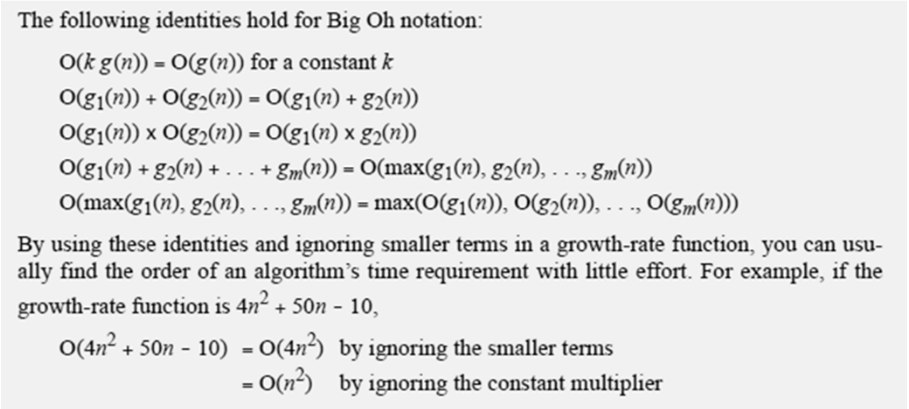
- A function f(n) is of order at most g(n)

- That is, f(n) is O(g(n))—if
  - A positive real number c and positive integer N exist …
  - Such that $f(n) \leq c \times g(n)$ for all $n \geq N$
  - That is, $c \times g(n)$ is an upper bound on $f(n)$ when n is sufficiently large

Big Oh Notation



Big Oh Notation



The following identities hold for Big Oh notation:

$O(k\, g(n)) = O(g(n))$ for a constant k

$O(g_1(n)) + O(g_2(n)) = O(g_1(n) + g_2(n))$

$O(g_1(n)) \times O(g_2(n)) = O(g_1(n) \times g_2(n))$

$O(g_1(n) + g_2(n) + \ldots + g_m(n)) = O(\max(g_1(n), g_2(n), \ldots, g_m(n))$

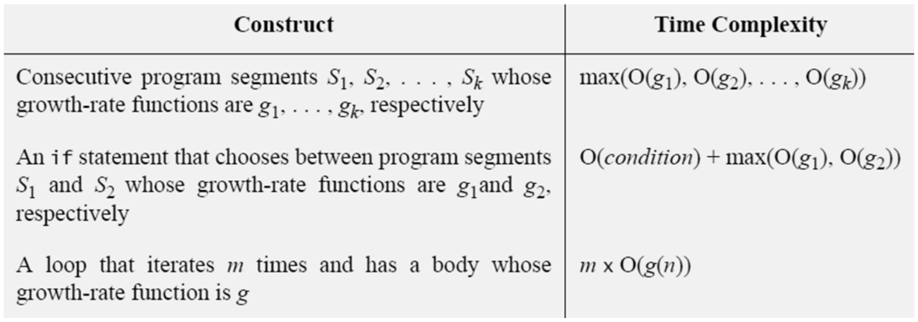
$O(\max(g_1(n), g_2(n), \ldots, g_m(n)) = \max(O(g_1(n)), O(g_2(n)), \ldots, O(g_m(n)))$

By using these identities and ignoring smaller terms in a growth-rate function, you can usually find the order of an algorithm's time requirement with little effort. For example, if the growth-rate function is $4n^2 + 50n - 10$,

$O(4n^2 + 50n - 10) = O(4n^2)$ by ignoring the smaller terms

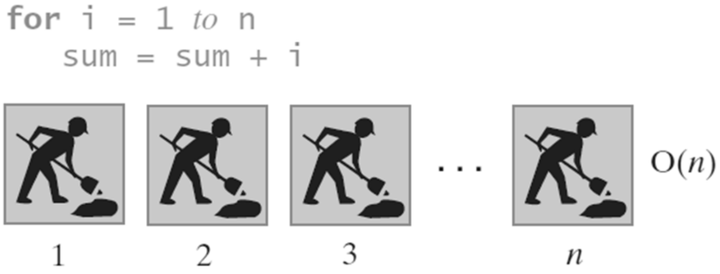
$= O(n^2)$ by ignoring the constant multiplier

Complexities of Program Constructs



Construct	Time Complexity
Consecutive program segments S_1, S_2, . . . , S_k whose growth-rate functions are g_1, . . . , g_k, respectively	max(O(g_1), O(g_2), . . . , O(g_k))
An `if` statement that chooses between program segments S_1 and S_2 whose growth-rate functions are $g_1$and g_2, respectively	O(*condition*) + max(O(g_1), O(g_2))
A loop that iterates *m* times and has a body whose growth-rate function is *g*	*m* x O($g(n)$)

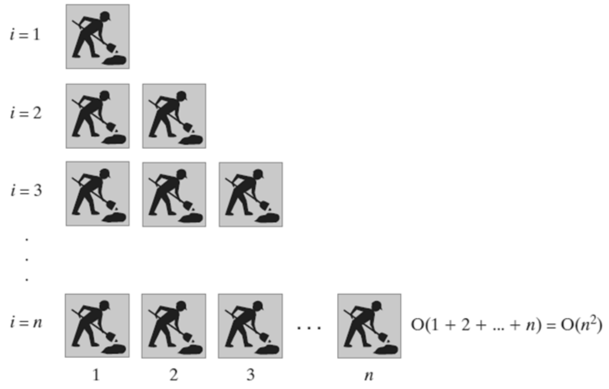
Picturing Efficiency – O(n)



```
for i = 1 to n
   sum = sum + i
```

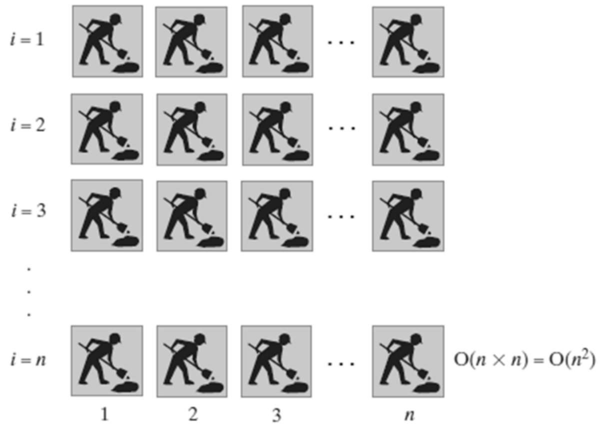
Picturing Efficiency – O(n^2)

```
for i = 1 to n
{  for j = 1 to i
       sum = sum + 1
}
```

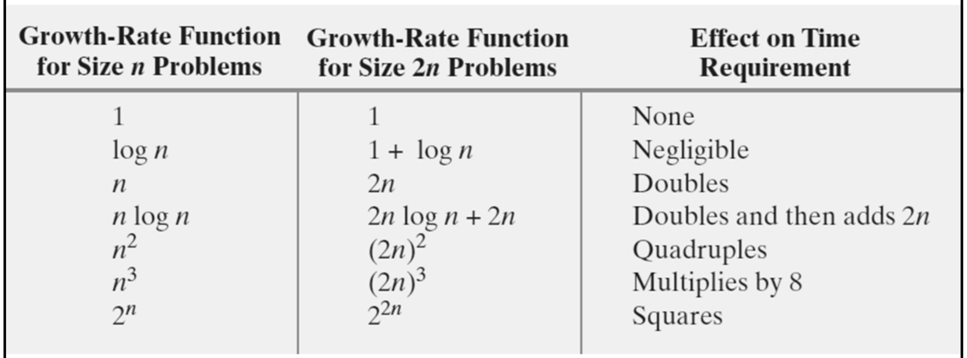


Picturing Efficiency – O(n^2)

```
for i = 1 to n
{  for j = 1 to n
       sum = sum + 1
}
```

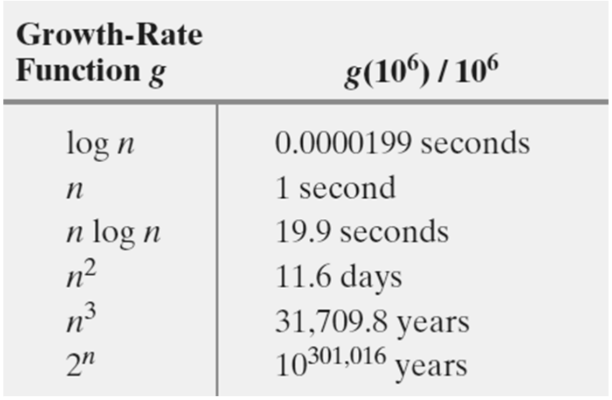


Effect of Doubling the Problem Size



Growth-Rate Function for Size n Problems	Growth-Rate Function for Size $2n$ Problems	Effect on Time Requirement
1	1	None
$\log n$	$1 + \log n$	Negligible
n	$2n$	Doubles
$n \log n$	$2n \log n + 2n$	Doubles and then adds $2n$
n^2	$(2n)^2$	Quadruples
n^3	$(2n)^3$	Multiplies by 8
2^n	2^{2n}	Squares

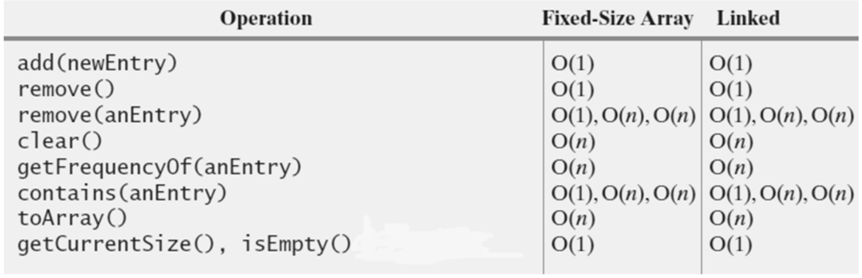
Time Required To Process One Million Items



Growth-Rate Function g	$g(10^6) / 10^6$
$\log n$	0.0000199 seconds
n	1 second
$n \log n$	19.9 seconds
n^2	11.6 days
n^3	31,709.8 years
2^n	$10^{301,016}$ years

Rate is one million operations per second

Efficiency of Implementations of ADT Bag



Operation	Fixed-Size Array	Linked
`add(newEntry)`	O(1)	O(1)
`remove()`	O(1)	O(1)
`remove(anEntry)`	O(1), O(*n*), O(*n*)	O(1), O(*n*), O(*n*)
`clear()`	O(*n*)	O(*n*)
`getFrequencyOf(anEntry)`	O(*n*)	O(*n*)
`contains(anEntry)`	O(1), O(*n*), O(*n*)	O(1), O(*n*), O(*n*)
`toArray()`	O(*n*)	O(*n*)
`getCurrentSize(), isEmpty()`	O(1)	O(1)