11 Problems: LU Decomposition

1. Consider the linear system:

$$\begin{aligned} x^1 &= v^1 \\ l_1^2 x^1 + x^2 &= v^2 \\ \vdots &\quad \vdots \\ l_1^n x^1 + l_2^n x^2 + \cdots + x^n &= v^n \end{aligned}$$

   i. Find x^1.

   ii. Find x^2.

   iii. Find x^3.

   k. Try to find a formula for x^k. Don't worry about simplifying your answer.

2. Let $M = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$ be a square $n \times n$ block matrix with W invertible.

   i. If W has r rows, what size are X, Y, and Z?

   ii. Find a UDL decomposition for M. In other words, fill in the stars in the following equation:

$$\begin{pmatrix} X & Y \\ Z & W \end{pmatrix} = \begin{pmatrix} I & * \\ 0 & I \end{pmatrix} \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \begin{pmatrix} I & 0 \\ * & I \end{pmatrix}$$