Problems for Gaussian Elimination

1. State whether the following augmented matrices are in RREF and compute their solution sets.

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right),$$

$$\left(\begin{array}{cccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right),$$

$$\left(\begin{array}{ccccccc|c} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right).$$

2. Show that this pair of augmented matrices are row equivalent, assuming $ad - bc \neq 0$:

$$\left(\begin{array}{cc|c} a & b & e \\ c & d & f \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{de-bf}{ad-bc} \\ 0 & 1 & \frac{af-ce}{ad-bc} \end{array}\right)$$

3. Consider the augmented matrix: $\left(\begin{array}{cc|c} 2 & -1 & 3 \\ -6 & 3 & 1 \end{array}\right)$

Give a *geometric* reason why the associated system of equations has no solution. (Hint, plot the three vectors given by the columns of this augmented matrix in the plane.) Given a general augmented matrix

$$\left(\begin{array}{cc|c} a & b & e \\ c & d & f \end{array}\right),$$

can you find a condition on the numbers a, b, c and d that create the geometric condition you found?

4. List as many operations on augmented matrices that *preserve* row equivalence as you can. Explain your answers. Give examples of operations that break row equivalence.

5. Row equivalence of matrices is an example of an *equivalence relation*. Recall that a relation $\sim$ on a set of objects U is an equivalence relation if the following three properties are satisfied:

   - Reflexive: For any $x \in U$, we have $x \sim x$.
   - Symmetric: For any $x, y \in U$, if $x \sim y$ then $y \sim x$.
   - Transitive: For any x, y and $z \in U$, if $x \sim y$ and $y \sim z$ then $x \sim z$.

   (For a fuller discussion of equivalence relations, see Homework 0, Problem 4)

   Show that row equivalence of augmented matrices is an equivalence relation.