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Multiplicative Closure

$P_3^{\mathbb{R}} = \{ax^3 + bx^2 + cx + d \mid a, b, c, d \in \mathbb{R}\}$

let $p \in P_3^{\mathbb{R}}$, $k \in \mathbb{R}$

$p = a_3x^3 + a_2x^2 + a_1x^1 + a_0$

$k \cdot p = k\,(a_3x^3 + a_2x^2 + a_1x^1 + a_0)$

$\boxed{= ka_3x^3 + ka_2x^2 + ka_1x^1 + ka_0}$

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9/29/15
Professor Stone

5) $P_3^{\mathbb{R}} = \left\{ ax^3 + bx^2 + cx + d \;\middle|\; a, b, c, d \in \mathbb{R} \right\}$

Additivity Zero: $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ ✓

Additive Inverse: $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} + \begin{pmatrix} -a \\ -b \\ -c \\ -d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ✓

Unity

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \cdot 1 = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Distributivity with 2 scalars: $k, h \in \mathbb{R}$

$$(k+h) \cdot (ax^3 + bx^2 + cx + d)$$

$$\downarrow \quad ax^3(k+h) + bx^2(k+h) + cx(k+h) + d(k+h)$$

$$\downarrow \quad \underbrace{ax^3k + ax^3h + bx^2k + bx^2h + cxk + cxh + dk + dh}_{\text{L.H.S.}}$$

$$k \cdot (ax^3 + bx^2 + cx + d) + h \cdot (ax^3 + bx^2 + cx + d)$$

$$\downarrow \quad \underbrace{kax^3 + kbx^2 + kcx + kd + hax^3 + hbx^2 + hcx + dh}_{\text{RHS.}}$$

So, we conclude the LHS = RHS

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Additive Closure

Let The Cooeficients of: ax^3+bx^2+cx+d be a vector. Thus:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}.$$

Take $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} + \begin{pmatrix} e \\ F \\ g \\ H \end{pmatrix} = \begin{pmatrix} a+e \\ b+f \\ c+g \\ d+h \end{pmatrix} \in \mathbb{R}$

Which would simplify to

$$(a+c)x^3 + (b+f)x^2 + (c+g)x + (d+h) \in P_3^{\mathbb{R}}$$

which Satisfys addition closure

Additive Commutativity.

$a, b, c, d, e, f, g, h \in \mathbb{R}$

$(ax^3+bx^2+cx+d)+(ex^3+fx^2+gx+h)=$
$x^3(a+e)+x^2(b+f)+x(c+g)+(d+h)$

$(ex^3+fx^2+gx+h)+(ax^3+bx^2+cx+d)=$
$x^3(e+a)+x^2(f+b)+x(g+c)+(h+d)$

• iv) Associativity

$V = \{ f \mid f: N \to R, \lim_{n \to \infty} f \in R \}$

$P_3^R = \{ ax^3 + bx^2 + cx + d \mid a, b, c, d \in R \}$

$(cd)V = c(dV)$

$(cd)(ax^3 + bx^2 + cx + d) = c(d(ax^3 + bx^2 + cx + d)$

$= c(ax^3d + bx^2d + cxd + dd)$

$(ax^3cd + bx^2cd + cxcd + dcd) = (ax^3cd + bx^2cd + cxcd + dcd]$

Xuchen Ji
Ke Xu
Nicolas Gomez

Distributivity (2 vectors & 1 scalar)

Let

$k \in \mathbb{R},\ (a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0)\ \&\ (b_3x^3 + b_2x^2 + b_1x^1 + b_0x^0) \in V$

$k(p+q) = kp + kq$

$p = (a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0)$

$q = (b_3x^3 + b_2x^2 + b_1x^1 + b_0x^0)$

<u>Left</u>: $k((a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0) + (b_3x^3 + b_2x^2 + b_1x^1 + b_0x^0))$

$\hookrightarrow k(a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0 + b_3x^3 + b_2x^2 + b_1x^1 + b_0x^0)$

$\Rightarrow kx^3(a_3 + b_3) + kx^2(a_2 + b_2) + kx(a_1 + b_1) + kx^0(a_0 + b_0)$

<u>Right</u>: $k(a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0) + k(b_3x^3 + b_2x^2 + b_1x^1 + b_0x^0)$

$\hookrightarrow ka_3x^3 + ka_2x^2 + ka_1x^1 + ka_0x^0 + kb_3x^3 + kb_2x^2 + kb_1x^1 + kb_0x^0$

$\Rightarrow kx^3(a_3 + b_3) + kx^2(a_2 + b_2) + kx(a_1 + b_1) + kx^0(a_0 + b_0)$