Math 253 **Exam 2** **11/18/2015**

Name: ______________________________

Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Sometimes/Always/Never questions. For all other questions, please circle your answers and justify your work for full credit. There are 13 questions for a total of 100 points.

Sometimes/Always/Never: Read the statement and decide whether the statement is sometimes true, always true, or never true. No work is necessary.

_______ 1. (5 points) If M is a square matrix and t a real number, then $\det(tM) = t \cdot \det(M)$.

A. Sometimes B. Always C. Never

_______ 2. (5 points) Let a be a real number. Then the matrix $\begin{pmatrix} 1 & -\frac{2}{5} & 101 & -2.35 \\ 0 & \frac{4}{7} & 0.56 & 35 \\ 0 & 0 & a & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is invertible.

A. Sometimes B. Always C. Never

_______ 3. (5 points) The vectors $(\frac{1}{32}, 201, -3)$, $(\frac{1}{14}, \frac{1}{101}, 0)$, $(0, \frac{15}{51}, \frac{101}{17})$, and $(\frac{21}{2}, 0, -1)$ form a basis of $\mathbb{R}^3$.

A. Sometimes B. Always C. Never

_______ 4. (5 points) Assume that A and B are $n \times n$ matrices. If A is not invertible and B is invertible, then AB is invertible.

A. Sometimes B. Always C. Never

_______ 5. (5 points) If A and B are $n \times n$ matrices, then $\det(A + B) = \det(A) + \det(B)$.

A. Sometimes B. Always C. Never

_______ 6. (5 points) The union of two subspaces of a vector space V is always a subspace of V.

A. Sometimes B. Always C. Never

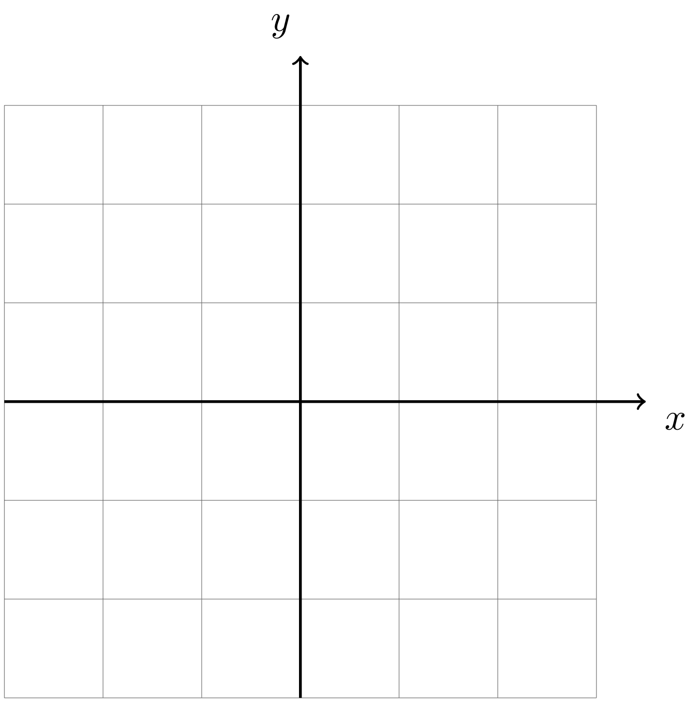
Short Answer. Make sure and justify your answer for full credit.

7. (10 points) Let $S = \{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} = (r, 3r, 5r),\ r \in \mathbb{R}\}$. Show that S is a subspace of $\mathbb{R}^3$.

8. (5 points) Show that $(4, 2, 1)$ is an element of the the set $\text{span}\{(1, 2, -1), (3, 1, 2)\}$.

9. (5 points) Write $(2, 3)$ as a linear combination of the vectors $(1, 3)$ and $(-4, 7)$.

10. (10 points) Let $\mathbf{u} = (1, 0)$ and $\mathbf{v} = (0, 1)$ be vectors in $\mathbb{R}^2$. If L is a linear transformation such that $L\mathbf{u} = (1, 1)$, $L\mathbf{v} = (-1, 1)$, then use a geometric argument to determine whether or not $\mathbf{u} + \mathbf{v}$ is an eigenvector of L. Make sure and justify your reasons.



11. (a) (5 points) Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 0 & a & 3 \end{pmatrix}$.

(b) (5 points) What are the eigenvalues of the matrix A?

(c) (5 points) Let $a = 0$ and find the eigenspace for each eigenvalue of A. Write your answer in span notation.

12. (10 points) Consider the vectors $(1,-1,0)$, $(0,1,4)$, and $(2,-30,h+4)$. For what values of h are the vectors linearly independent? Justify your answer.

13. (a) (10 points) Show that $B = \{(1,0,0),(1,1,0),(1,1,1)\}$ is a basis for $\mathbb{R}^3$.

(b) (5 points) Consider the linear transformation $T(x,y,z) = (x - y + z, z - x)$. Find the matrix representation of T with respect to the basis $B = \{(1,0,0),(1,1,0),(1,1,1)\}$.