

Name: \_\_\_\_\_

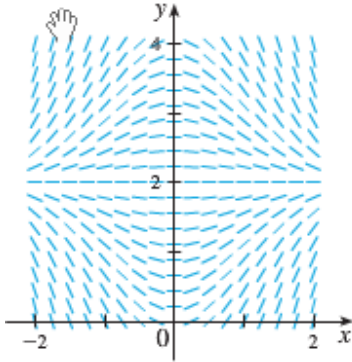
Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the True/False or the Multiple Choice questions. For all other questions, please circle your answers and show your work for full credit. There are 14 questions for a total of 100 points.

**True or False: Please circle either true or false. No work is necessary.**

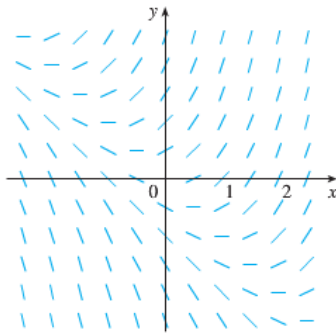
- \_\_\_\_\_ 1. (5 points)  $\lim_{n \rightarrow \infty} \sum_{i=0}^n x_i^2 \Delta x = \frac{1}{3}$  where  $\{x_i\}$  is a partition of the interval  $[0, 1]$  for each  $n$ .  
A. True B. False
- \_\_\_\_\_ 2. (5 points) Euler's method is a numerical procedure that uses the tangent lines to approximate roots of a polynomial.  
A. True B. False
- \_\_\_\_\_ 3. (5 points) If the series  $\sum c_n x^n$  diverges when  $x = 6$ , then the series diverges when  $x = -10$ .  
A. True B. False
- \_\_\_\_\_ 4. (5 points) If  $f$  is continuous, then  $\int_{-\infty}^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x) dx$ .  
A. True B. False

**Matching.** For each question match **exactly one** item one group with **exactly one** item from the other group.

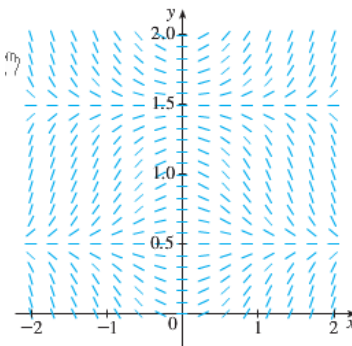
5. (5 points) Match the direction fields on the left with the differential equations on the right.



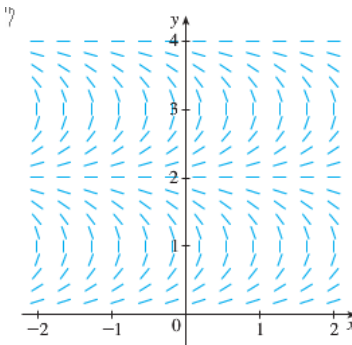
A.  $y' = x \cos(\pi y)$



B.  $y' = \sin x \sin y$



C.  $y' = \tan\left(\frac{1}{2}\pi y\right)$



D.  $y' = x + y$

**Short Answer: Show your work for full credit.**

6. (5 points) Evaluate  $\int \frac{r}{1+3r^2} dr$

7. (5 points) Evaluate  $\int x \sin(x) dx$

8. (5 points) Write the following rational function as a partial fraction:  $\frac{1}{x^2 + 2x + 5}$

9. (5 points) Evaluate  $\int_{-3}^2 |x - 2| dx$

10. (5 points) Evaluate  $\int_1^e x^6 \ln(x) dx$

11. (10 points) The Laplace Transform of  $f$  is the function  $\mathcal{L}f(s)$  of the variable  $s$  defined by the improper integral (if it converges)

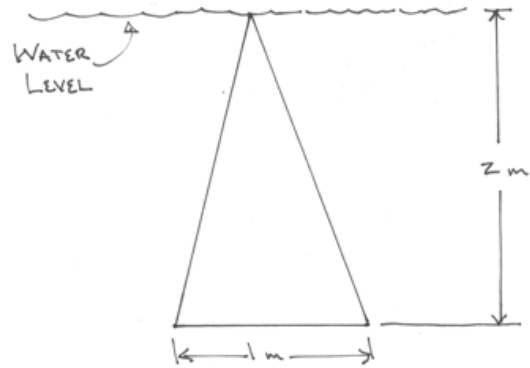
$$\mathcal{L}f(s) = \int_0^{\infty} f(x)e^{-sx} dx.$$

Show that if  $f(x) = C$ , where  $C$  is a constant, then  $\mathcal{L}f(x) = C/s$ .

12. (10 points) Find the function  $f$  such that  $f'(x) = f(x)(1 - f(x))$  and  $f(0) = 1/2$ .

13. (15 points) Recall that pressure at a depth  $h$  in a liquid of density  $\rho$  is given by the equation:  $p = \rho gh$ . For a given depth, the fluid force is equal to the pressure times the surface area of the object.

In this problem, a plate the shape of an isosceles triangle with base 1 m and height 2 m is submerged vertically in a tank of water. Set up (but do not compute) the integral that represents the fluid force against the triangular region.



14. (15 points) Recall that work equals force times the distance traveled. Set up (but do not compute) the integral that represents the work (against gravity) required to build a concrete column of height 5 m and square base of side 2 m. Assume that concrete has density  $1500 \text{ kg/m}^3$ . Justify your answer.