“Limit” from Diderot’s *Encyclopédie*[*]

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LIMIT, n.n. (Math.) We say that one magnitude is the *limit* of another magnitude when the second may approach the first by less than any given magnitude, so small that we may suppose that the magnitude which approaches may never exceed the magnitude which it approaches; so that the difference between such a quantity and its *limit* is absolutely unassignable.

For example, let us suppose there are two polygons, one inscribed in, and the other circumscribed about, a cicrle. It is evident that we may increase the number of sides to as many as we may wish and, in this case, each polygon will come closer and closer to the circumference of the circle, the perimeter of the inscribed polygon increasing, and that of the circumscribed decreasing. However, the perimeter of the first will never surpass the circumference, nor will the second ever be smaller than the same circumference. Thus, the circumference is the *limit* of the augmentation of the first polygon, and of the dimunition of the second.

1. If two magnitudes are the limits of the same quantity, these two magnitudes will be equal to one another.

2. Let $A \times B$ be the product of two magnitudes A and B. Let us suppose that C be the *limit* of the magnitude A and D the *limit* fo the quantity B. I say that $C \times D$, the product of the *limits*, will necessarily be the *limit* of $A \times B$, the product of the two magnitudes A and B.

[*]Translated from the original French text by Rob Bradley, Department of Mathematics and Computer Science, Adelphi University. Prepared for ARITHMOS V, the meeting of June 29, 2002; for more information, please visit `www.arithmos.org`.

These two propositions, which are precisely demonstrated in the *Insitutions de Géometrie*[1], serve as principles in the rigorous demonstration that the area of a circle is the product of the semi-circumference and the radius. *See* the cited work, *p. 331 ff of the second volume.* (E)

The theory of *limits* is the basis of the true metaphysics of the differential calculus. *See* **Differential**, **Fluxion**, **Exhaustion**, **Infinite**. Properly speaking, the *limit* never coincides, or never becomes equal to, the quantity of which it is the *limit*, but it always approaches, closer and closer, and may differ by as little as we wish. The circle, for example, is the *limit* of inscribed and circumscribed polygons, for it never coincides exactly with them, although these may approach it at infinity. This notion may serve to clarify many mathematical propositions. Fro example, the sum of a decreasing geometric sequence whose first term is a and whose second is b is $\frac{a-b}{aa}$. This value is not, properly speaking, the sum of the progression, it is the *limit* of that sum; that is, the quantity which we may approach as near as we wish, without ever actually achieving. For if e is the last term of the progression, the exact value of the sum is $\frac{aa-be}{a-b}$, which is always less that $\frac{aa}{a-b}$, since even in a decreasing geometric series, the final term e is never $= 0$. However, since this term continuously approaches zero, without ever arriving, it is clear that zero is the *limit*. As a consequence, the *limit* of $\frac{aa-be}{a-b}$ is $\frac{aa}{a-b}$ by supposing that $e = 0$; that is, by substituting for e its *limit*. *See* **Sequence** *or* **Series**, **Progression**, *etc.* (O)

[1]This textbook was written by de la Chapelle himself.