Comment on "Quantum Interferometric Optical Lithography: Exploiting Entanglement to Beat the Diffraction Limit"

In a recent Letter, Boto *et al.* [1] show how to use entangled photons to write lithographic gratings with a resolution that exceeds the classical Rayleigh criterion by the factor N, where N is the number of entangled photons and where the recording medium operates by N-photon absorption. In the special case $N = 2$ the method entails generating entangled photon pairs through parametric down-conversion in a nonlinear crystal, combining these waves at a symmetric beam splitter, and interfering these beams at a two-photon absorbing medium (see Fig. 1). Quantum interference effects require the two photons of the photon pair to emerge either both in the upper arm (b_2) or both in the lower arm (a_2), but never one photon in each arm. The two-photon excitation rate depends on the interference of the probability amplitudes for each of the arms, leading to an interference pattern of the form $1 + \cos 2\phi$, where ϕ is the classical (one-photon) phase difference between the paths. In contrast, the classical interference pattern has the form $1 + \cos\phi$.

An anticipated experimental challenge in implementing this proposal is the conflicting needs that the light field be sufficiently weak to contain only two photons per mode yet be sufficiently strong to excite two-photon absorption. In this Comment, we suggest a variation of the method that allows the use of much more intense light fields yet still produces sub-Rayleigh fringes. We propose replacing the parametric down-converter with a high-gain parametric amplifier, which is basically the same physical device but pumped by a much stronger laser field. The output of a high-gain parametric amplifier consists of two intense, strongly correlated light fields, which, however, do not consist of photon pairs, the key ingredient of the proposal of Boto *et al.*

We describe the light field leaving the parametric amplifier by photon operators $\hat{a}_1 = U\hat{a}_0 + V\hat{b}_0^\dagger$ and $\hat{b}_1 = U\hat{b}_0 + V\hat{a}_0^\dagger$, where $\hat{a}_0$ and $\hat{b}_0$ represent the input fields and U and V represent the transfer characteristics of the amplifier, which we express as $U = \cosh G$, $V = -i\exp(i\delta)\sinh G$, where G is the single-pass gain and δ is a relative phase. The gain factor G may be written as $G = g|E_p|L$ where L is the interaction path length, $|E_p|$ is the pump laser amplitude, and g is a gain coefficient proportional to the nonlinear susceptibility $\chi^{(2)}$. We take the transfer characteristics of the beam splitter to be $\hat{a}_2 = (\hat{a}_1 - i\hat{b}_1)/\sqrt{2}$ and $\hat{b}_2 = (-i\hat{a}_1 + \hat{b}_1)/\sqrt{2}$. We then take the total field at the recording medium as $\hat{a}_3 = -\hat{a}_2\exp(i\phi) - \hat{b}_2$, where ϕ varies with position, or explicitly as

$$\hat{a}_3 = (1/\sqrt{2})[(i - e^{i\phi})(U\hat{a}_0 + V\hat{b}_0^\dagger) + (ie^{i\phi} - 1)(U\hat{b}_0 + V\hat{a}_0^\dagger)]. \quad (1)$$

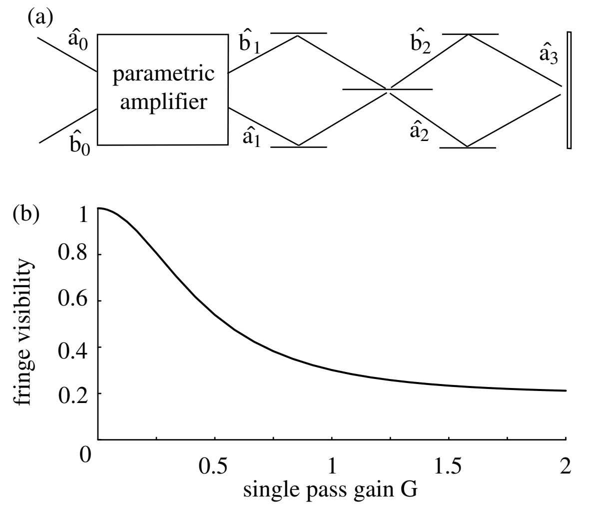

FIG. 1. (a) Setup for the recording of sub-Rayleigh fringes using entangled states of light. (b) Variation of the two-photon fringe visibility with single-pass gain.

We calculate the two-photon transition probability (for input fields $\hat{a}_0$ and $\hat{b}_0$ in the vacuum state) as $\langle \hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 \rangle = 4|V|^2[\cos^2\phi + |V|^2(2 + \cos^2\phi)]$. From this result we readily find that the visibility of the resulting two-photon excitation pattern is given by $(1 + |V|^2)/(1 + 5|V|^2)$. The functional dependence of the visibility on the single-pass gain G is shown in the figure. We see that the fringe visibility drops from unity at small gain to an asymptotic value of $1/5$ at high gain. The drop in visibility occurs because at high gain more than one photon pair is created, and two-photon absorption can occur from photons coming both from $\hat{a}_1$ or both from $\hat{b}_1$. A fringe visibility of 20% is more than adequate for applications such as holography. Our modification to the proposal of Boto *et al.* is likely to help greatly with its laboratory implementation.

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[1] A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. **85**, 2733 (2000).

