True or False: Please circle either true or false. No work is necessary.

1. (5 points) If \( \{a_n\} \) is a decreasing sequence and \( a_n > 0 \) for all \( n \), then \( \{a_n\} \) is convergent.
   A. True   B. False

2. (5 points) If \( f(x) = 2(x - 1) - (x - 1)^2 + \frac{1}{3}(x - 1)^3 - \cdots \) is convergent for all values of \( x \), then \( f'''(1) = 3 \).
   A. True   B. False

3. (5 points) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e} \).
   A. True   B. False

4. (5 points) If the series \( \sum c_n x^n \) diverges when \( x = 6 \), then the series diverges when \( x = -10 \).
   A. True   B. False

Multiple Choice: Please circle your answer. No work is necessary, but partial credit will be given if work is shown.

5. (5 points) If the limit of the sequence \( a_n \) defined by \( a_{n+1} = -\frac{4}{4 + a_n} \) exists, then the limit is
   A. 1
   B. -1
   C. 2
   D. -2
   E. \( \pi \)
6. (5 points) Which of the following series are divergent? (There might be more than one.)

A. \[ \sum_{n=1}^{\infty} \frac{n^2 + 4n - 1}{\sqrt{n^5 + \pi n + 9}} \]
B. \[ \sum_{n=1}^{\infty} \frac{n + 1}{n^4} \]
C. \[ \sum_{n=1}^{\infty} \frac{1}{\pi^n} \]
D. \[ \sum_{n=1}^{\infty} \frac{e^n + 1}{e^{2n}} \]
E. \[ \sum_{n=1}^{\infty} \frac{1}{n(n - 1)} \]

7. (5 points) The radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n} \) is

A. 0
B. 5
C. \( \infty \)
D. \( \frac{1}{5} \)

8. (5 points) The Taylor series of \( \sin(x^2) \) centered at \( a = 0 \) is

A. \[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n + 1)!} \]
B. \[ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \]
C. \[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!} \]
D. \[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n + 1)!} \]
9. (5 points) Let $T_3(x)$ be the degree 3 taylor polynomial of $\sin(x)$ at $a = 0$. Using Taylor’s inequality, the bound for $|R_3(x)| = |\sin(x) - T_3(x)|$ for $x \in [0, 0.1]$ is

A. $\frac{1}{3!}(0.1)^3$  
B. $\frac{x^4}{3!}$  
C. $\frac{1}{4!}(x)^4$  
D. $\frac{1}{4!}(0.1)^4$

10. (5 points) $\sum_{n=1}^{\infty} 2^{2n} 5^{1-n}$ is

A. convergent and equal to 5  
B. convergent and equal to 5/4  
C. convergent and equal to 20  
D. divergent and equal to $\infty$  
E. none of the above

11. (5 points) Which of the following statements are correct?

I. Every convergent series is absolutely convergent.
II. If a series is absolutely convergent, then it is convergent.
III. The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ is absolutely convergent.

A. I,II and III  
B. I and III  
C. II and III  
D. only III  
E. I and II
Short Answer: Show your work for full credit.

12. (5 points) Use series to evaluate the limit \( \lim_{x \to 0} \frac{\sin(x) - x}{x^3} \).

13. (a) (5 points) Explain why the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5}{n^5} \) is convergent.

(b) (5 points) Find the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} \) to two decimal places.
14. (15 points) Find the interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n} \).
15. (a) (10 points) Find the power series representation of \( \frac{1}{(1-x)^2} \) (HINT: \( \frac{d}{dx} \left( \frac{1}{1-x} \right) = \frac{-1}{(1-x)^2} \)).

(b) (5 points) What is the radius of convergence of the series in part (a)?