

# Systems I: Computer Organization and Architecture

## Lecture 5: Quine McCluskey Algorithm

### Why Use the Quine McCluskey Algorithm?

- While Karnaugh maps provide an easy method for simplifying the functions that we have seen so far, they do not work well when there are more than six independent variables.
- One common method is the Quine McCluskey Algorithm, which makes extensive use of the axiom  $\mathbf{xy' + xy = x}$  to simplify terms.

## The Basic Steps in the Algorithm

1. Write the function  $F$  as the sum of minterms
2. By using the rule  $\mathbf{xy' + xy = x}$ , eliminate as many terms as possible to produce the prime implicants.
3. Use a prime implicant charts to eliminate redundant terms.

## Rewriting and Combining Minterms

- We start out by rewriting minterms as binary numbers:

E.g.,

$w'x'yz$  ( $m_3$ ) becomes 0011

$w'x'y'z$  ( $m_1$ ) 0001

These combine to form 00-1  $w'x'z$

- We can only combine them if they differ by *exactly 1 bit*.

## Example: Simplifying $\Sigma(0,4,5,7,8,11,12,15)$

- We write the minterms in binary form, grouping them by the number of ones in the binary form.

$m_0$	<u>0000</u>	0 1's
$m_4$	0100	1 1's
$m_8$	<u>1000</u>	
$m_5$	0101	2 1's
$m_{12}$	<u>1100</u>	
$m_7$	0111	3 1's
$m_{11}$	<u>1011</u>	
$m_{15}$	1111	4 1's

## Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We now look for minterms whose binary form differs with only one bit:

$m_0$	<u>0000</u>	←	(0,4)	0-00
$m_4$	0100	←		
$m_8$	<u>1000</u>			
$m_5$	0101			
$m_{12}$	<u>1100</u>			
$m_7$	0111			
$m_{11}$	<u>1011</u>			
$m_{15}$	1111			

## Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We check the minterms for which there is a simplification:

$m_0$	$\frac{\sqrt{0000}}{\quad}$	(0,4) 0-00
$m_4$	$\frac{\sqrt{0100}}{\quad}$	
$m_8$	$\frac{1000}{\quad}$	
$m_5$	0101	
$m_{12}$	$\frac{1100}{\quad}$	
$m_7$	0111	
$m_{11}$	$\frac{1011}{\quad}$	
$m_{15}$	1111	

## Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\frac{\sqrt{0000}}{\quad}$	(0,4) 0-00
$m_4$	$\frac{\sqrt{0100}}{\quad}$	(0,8) -000
$m_8$	$\frac{\sqrt{1000}}{\quad}$	
$m_5$	0101	
$m_{12}$	$\frac{1100}{\quad}$	
$m_7$	0111	
$m_{11}$	$\frac{1011}{\quad}$	
$m_{15}$	1111	

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4,5)	010-
$m_5$	$\sqrt{0101}$		
$m_{12}$	$\underline{1100}$		
$m_7$	0111		
$m_{11}$	$\underline{1011}$		
$m_{15}$	1111		

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4,5)	010-
$m_5$	$\sqrt{0101}$	(4,12)	-100
$m_{12}$	$\sqrt{1100}$		
$m_7$	0111		
$m_{11}$	$\underline{1011}$		
$m_{15}$	1111		

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4, 5)	010-
$m_5$	$\sqrt{0101}$	(4,12)	-100
$m_{12}$	$\sqrt{1100}$	(8,12)	1-00
$m_7$	0111		
$m_{11}$	<u>1011</u>		
$m_{15}$	1111		

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4, 5)	010-
$m_5$	$\sqrt{0101}$	(4,12)	-100
$m_{12}$	$\sqrt{1100}$	(8,12)	1-00
$m_7$	$\sqrt{0111}$	(5, 7)	01-1
$m_{11}$	<u>1011</u>		
$m_{15}$	1111		

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4, 5)	010-
$m_5$	$\sqrt{0101}$	(4,12)	-100
$m_{12}$	$\sqrt{1100}$	(8,12)	1-00
$m_7$	$\sqrt{0111}$	(5, 7)	01-1
$m_{11}$	$\sqrt{1011}$	(7, 15)	-111
$m_{15}$	$\sqrt{1111}$		

### Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

$m_0$	$\sqrt{0000}$	(0,4)	0-00
$m_4$	$\sqrt{0100}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(4, 5)	010-
$m_5$	$\sqrt{0101}$	(4,12)	-100
$m_{12}$	$\sqrt{1100}$	(8,12)	1-00
$m_7$	$\sqrt{0111}$	(5, 7)	01-1
$m_{11}$	$\sqrt{1011}$	(7, 15)	-111
$m_{15}$	$\sqrt{1111}$	(11, 15)	1-11

## Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- We now do the same thing to the newly simplified terms:

$m_0$	$\sqrt{0000}$	$\sqrt{(0,4)}$	$0-00$	$\leftarrow (0,4,8,12)$	$--00$
$m_4$	$\sqrt{0100}$	$(0,8)$	$-000$		
$m_8$	$\sqrt{1000}$	$(4,5)$	$010-$		
$m_5$	$\sqrt{0101}$	$(4,12)$	$-100$		
$m_{12}$	$\sqrt{1100}$	$\sqrt{(8,12)}$	$1-00$		
$m_7$	$\sqrt{0111}$	$(5,7)$	$01-1$		
$m_{11}$	$\sqrt{1011}$	$(7,15)$	$-111$		
$m_{15}$	$\sqrt{1111}$	$(11,15)$	$1-11$		

## Simplifying Terms in $\Sigma(0,4,5,7,8,11,12,15)$

- These new terms may produce simplified terms that are copies of each other:

$m_0$	$\sqrt{0000}$	$\sqrt{(0,4)}$	$0-00$	$(0,4,8,12)$	$--00$
$m_4$	$\sqrt{0100}$	$\sqrt{(0,8)}$	$-000$	$\leftarrow (0,8,4,12)$	$--00$
$m_8$	$\sqrt{1000}$	$(4,5)$	$010-$		
$m_5$	$\sqrt{0101}$	$\sqrt{(4,12)}$	$-100$		
$m_{12}$	$\sqrt{1100}$	$\sqrt{(8,12)}$	$1-00$		
$m_7$	$\sqrt{0111}$	$(5,7)$	$01-1$		
$m_{11}$	$\sqrt{1011}$	$(7,15)$	$-111$		
$m_{15}$	$\sqrt{1111}$	$(11,15)$	$1-11$		



## The Terms Left In $\Sigma(0,4,5,7,8,11,12,15)$

- We now have:

(0,4,8,12) --00

~~(0,8,4,12) --00~~

*Duplicate term*

(4, 5) 010-

(5, 7) 01-1

(7, 15) -111

(11, 15) 1-11

$$F = y'z' + w'xy' + w'xz + xyz + wyz$$

There is still some duplicate even after finding the prime implicants. We need the Prime Implicant Table to eliminate the unnecessary terms

## Prime Implicant Table

PI	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
$xyz$	-111				X				X
$wyz$	1-11						X		X

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
$xyz$	-111				X				X
$wyz$	1-11						X		X

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
$xyz$	-111				X				X
$wyz$	1-11						X		X

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
$xyz$	-111				X				X
$wyz$	1-11						X		X

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	X	X			X		X	
$w'xy'$	010-		X	X					
$w'xz$	01-1			X	X				
$xyz$	-111				X				X
$wyz$	1-11						X		X

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
$xyz$	-111				x				x
$wyz$	1-11						x		x

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
$xyz$	-111				x				x
$wyz$	1-11						x		x

## Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
$xyz$	-111				x				x
$wyz$	1-11						x		x

## Prime Implicant Table

<u>PI</u>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
$xyz$	-111				x				x
$wyz$	1-11						x		x

## Prime Implicant Table

<b>PI</b>	Numeric	0	4	5	7	8	11	12	15
$y'z'$	--00	x	x			x		x	
$w'xy'$	010-		x	x					
$w'xz$	01-1			x	x				
$xyz$	-111				x				x
$wyz$	1-11						x		x

$$F = y'z' + w'xz + wyz$$

### Example: $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

- $m_0$  0000 0 1's
- $m_1$  0001 1 1's
- $m_2$  0010
- $m_8$  1000
- $m_5$  0101 2 1's
- $m_6$  0110
- $m_9$  1001
- $m_{10}$  1010
- $m_7$  0111 3 1's
- $m_{14}$  1110

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\frac{\sqrt{0000}}$	←	(0,1)	000-
$m_1$	$\frac{\sqrt{0001}}$	←		
$m_2$	0010			
$m_8$	$\frac{1000}{\quad}$			
$m_5$	0101			
$m_6$	0110			
$m_9$	1001			
$m_{10}$	$\frac{1010}{\quad}$			
$m_7$	0111			
$m_{14}$	1110			

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\frac{\sqrt{0000}}$	▼	(0,1)	000-
$m_1$	$\frac{\sqrt{0001}}$		(0,2)	00-0
$m_2$	$\frac{\sqrt{0010}}$	←		
$m_8$	$\frac{1000}{\quad}$			
$m_5$	0101			
$m_6$	0110			
$m_9$	1001			
$m_{10}$	$\frac{1010}{\quad}$			
$m_7$	0111			
$m_{14}$	1110			

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	<u><math>\sqrt{0000}</math></u>	$(0,1)$	000-
$m_1$	$\sqrt{0001}$	$(0,2)$	00-0
$m_2$	$\sqrt{0010}$	$(0,8)$	-000
$m_8$	<u><math>\sqrt{1000}</math></u>		
$m_5$	0101		
$m_6$	0110		
$m_9$	1001		
$m_{10}$	<u>1010</u>		
$m_7$	0111		
$m_{14}$	1110		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	<u><math>\sqrt{0000}</math></u>	$(0,1)$	000-
$m_1$	$\sqrt{0001}$	$(0,2)$	00-0
$m_2$	$\sqrt{0010}$	$(0,8)$	-000
$m_8$	<u><math>\sqrt{1000}</math></u>	$(1,5)$	0-01
$m_5$	<u><math>\sqrt{0101}</math></u>		
$m_6$	0110		
$m_9$	1001		
$m_{10}$	<u>1010</u>		
$m_7$	0111		
$m_{14}$	1110		



Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\sqrt{0000}$	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	0110		
$m_9$	$\sqrt{1001}$		
$m_{10}$	$\frac{1010}{\quad}$		
$m_7$	0111		
$m_{14}$	1110		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\sqrt{0000}$	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	$\sqrt{1010}$		
$m_7$	0111		
$m_{14}$	1110		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	<u><math>\sqrt{0000}</math></u>	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	<u><math>\sqrt{1000}</math></u>	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	<u><math>\sqrt{1010}</math></u>	(8,9)	100-
$m_7$	0111		
$m_{14}$	1110		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	<u><math>\sqrt{0000}</math></u>	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	<u><math>\sqrt{1000}</math></u>	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	<u><math>\sqrt{1010}</math></u>	(8,9)	100-
$m_7$	0111	(8,10)	10-0
$m_{14}$	1110		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\frac{\sqrt{0000}}$	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	$\frac{\sqrt{1000}}$	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	$\frac{\sqrt{1010}}$	(8,9)	100-
$m_7$	$\sqrt{0111}$	(8,10)	10-0
$m_{14}$	1110	(5,7)	01-1

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\frac{\sqrt{0000}}$	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	$\frac{\sqrt{1000}}$	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	$\frac{\sqrt{1010}}$	(8,9)	100-
$m_7$	$\sqrt{0111}$	(8,10)	10-0
$m_{14}$	1110	(5,7)	01-1
		(6,7)	011-

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$m_0$	$\sqrt{0000}$	(0,1)	000-
$m_1$	$\sqrt{0001}$	(0,2)	00-0
$m_2$	$\sqrt{0010}$	(0,8)	-000
$m_8$	$\sqrt{1000}$	(1,5)	0-01
$m_5$	$\sqrt{0101}$	(1,9)	-001
$m_6$	$\sqrt{0110}$	(2,6)	0-10
$m_9$	$\sqrt{1001}$	(2,10)	-010
$m_{10}$	$\sqrt{1010}$	(8,9)	100-
$m_7$	$\sqrt{0111}$	(8,10)	10-0
$m_{14}$	$\sqrt{1110}$	(5,7)	01-1
		(6,7)	011-
		(6,14)	-110
		(10,14)	1-10

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	$\leftarrow$	(0,1,8,9)	-00-
(0,2)	00-0			
(0,8)	-000			
(1,5)	0-01			
(1,9)	-001			
(2,6)	0-10			
(2,10)	-010			
$\sqrt{(8,9)}$	100-			
(8,10)	10-0			
(5,7)	01-1			
(6,7)	011-			
(6,14)	-110			
(10,14)	1-10			

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	← (0,2,8,10)	-0-0
(0,8)	<u>-000</u>		
(1,5)	0-01		
(1,9)	-001		
(2,6)	0-10		
(2,10)	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	← (0,8,1,9)	-00-
(1,5)	0-01		
$\sqrt{(1,9)}$	-001		
(2,6)	0-10		
(2,10)	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	(0,8,1,9)	-00-
(1,5)	0-01	(0,8,2,10)	-0-0
$\sqrt{(1,9)}$	-001		
(2,6)	0-10		
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
(10,14)	1-10		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	(0,1,8,9)	-00-
$\sqrt{(0,2)}$	00-0	(0,2,8,10)	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	(0,8,1,9)	-00-
(1,5)	0-01	(0,8,2,10)	-0-0
$\sqrt{(1,9)}$	-001	(2,6,10,14)	--10
$\sqrt{(2,6)}$	0-10		
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
(5,7)	01-1		
(6,7)	011-		
(6,14)	-110		
$\sqrt{(10,14)}$	1-10		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

$\sqrt{(0,1)}$	000-	$(0,1,8,9)$	-00-
$\sqrt{(0,2)}$	00-0	$(0,2,8,10)$	-0-0
$\sqrt{(0,8)}$	<u>-000</u>	$(0,8,1,9)$	-00-
$(1,5)$	0-01	$(0,8,2,10)$	-0-0
$\sqrt{(1,9)}$	-001	$(2,6,10,14)$	--10
$\sqrt{(2,6)}$	0-10	$(2,10,6,14)$	--10
$\sqrt{(2,10)}$	-010		
$\sqrt{(8,9)}$	100-		
$\sqrt{(8,10)}$	<u>10-0</u>		
$(5,7)$	01-1		
$(6,7)$	011-		
$\sqrt{(6,14)}$	-110		
$\sqrt{(10,14)}$	1-10		

Example:  $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

- These are the terms that we have left:

$(0,1,8,9)$	-00-	
$(0,2,8,10)$	-0-0	
<del><math>(0,8,1,9)</math></del>	<del>00</del>	← <i>Redundant</i>
<del><math>(0,8,2,10)</math></del>	<del>-0-0</del>	←
$(2,6,10,14)$	--10	
<del><math>(2,10,6,14)</math></del>	<del>--10</del>	←
$(1,5)$	<u>0-01</u>	
$(5,7)$	01-1	
$(6,7)$	011-	

## Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

## Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	<del>X</del>	<del>X</del>					<del>X</del>	<del>X</del>		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				



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PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

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PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

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PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

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$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

## Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	$\bar{x}$	$\bar{x}$					$\bar{x}$	$\bar{x}$		
$x'z'$	(0,2,8,10)	$\bar{x}$		$\bar{x}$				$\bar{x}$		$\bar{x}$	
$yz'$	(2,6,10,14)			$\bar{x}$		$\bar{x}$				$\bar{x}$	$\bar{x}$
$w'y'z$	(1,5)	$\bar{x}$	$\bar{x}$		$\bar{x}$						
$w'xz$	(5,7)				$\bar{x}$		$\bar{x}$				
$w'xy$	(6,7)					$\bar{x}$	$\bar{x}$				

## Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	$\bar{x}$	$\bar{x}$					$\bar{x}$	$\bar{x}$		
$x'z'$	(0,2,8,10)	$\bar{x}$		$\bar{x}$				$\bar{x}$		$\bar{x}$	
$yz'$	(2,6,10,14)			$\bar{x}$		$\bar{x}$				$\bar{x}$	$\bar{x}$
$w'y'z$	(1,5)	$\bar{x}$	$\bar{x}$		$\bar{x}$						
$w'xz$	(5,7)				$\bar{x}$		$\bar{x}$				
$w'xy$	(6,7)					$\bar{x}$	$\bar{x}$				

## Prime Implicants for $\Sigma(0,1, 2,5,6,7,8,9,10,14)$

PI	Numeric	0	1	2	5	6	7	8	9	10	14
$x'y'$	(0,1,8,9)	X	X					X	X		
$x'z'$	(0,2,8,10)	X		X				X		X	
$yz'$	(2,6,10,14)			X		X				X	X
$w'y'z$	(1,5)	X	X		X						
$w'xz$	(5,7)				X		X				
$w'xy$	(6,7)					X	X				

$$F = x'y' + yz' + w'xz$$