Math 390: Practice Problems for Quiz 1

Problems from the book

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Additional Problems

1. Determine whether the following graphs are isomorphic. Justify your answer.

![Graph 1](image1.png) ![Graph 2](image2.png)

2. (a) Draw all non-isomorphic simple graphs with 5 vertices and 9 edges.
   (b) Draw all non-isomorphic simple graphs with 5 vertices and 8 edges.
3. Consider the following graphs:

For each of these graphs, determine whether the graph is Eulerian. Justify your answers.

4. Consider the following graphs:

For each of these graphs, determine whether the graph is Hamiltonian. If the graph is Hamiltonian, indicate a Hamiltonian cycle.

5. (a) For which values of \( n \) is \( K_n \) Hamiltonian?
    (b) Which complete bipartite graphs are Hamiltonian?

6. Let \( G \) be a weighted graph with 5 vertices \( a, b, c, d, e \). For each edge, the weight is given by the following table (if two vertices do not share an edge, then the weight for those two vertices is given as \( \infty \)).

\[
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
 a & 0 & 3 & 5 & 11 & 9 \\
b & 3 & 0 & 3 & 9 & 8 \\
c & 5 & 3 & 0 & \infty & 10 \\
d & 11 & 9 & \infty & 0 & 7 \\
e & 9 & 8 & 10 & 7 & 0
\end{array}
\]

Find a minimum weight spanning tree for \( G \).

7. Consider the following graph \( G \):

Determine the connectivity \( \kappa(G) \) and the edge-connectivity \( \lambda(G) \) of this graph.
8. Draw a graph $G$ with $\kappa(G) = 2$ and $\lambda(G) = 3$.

9. Suppose that $G$ is a planar graph with 20 vertices, and suppose that every vertex of $G$ has degree 3. How many edges does $G$ have? How many faces does $G$ have?

10. Consider the following graphs:

For each of the above graphs, determine whether the graph is planar. Justify your answers.