Math 390 Homework 9

Due Friday, May 6

Solutions should be written in LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. (a) Let $G = (V_1, V_2)$ be a cubic bipartite graph. Show that there is a complete matching from $V_1$ to $V_2$ as well as a complete matching from $V_2$ to $V_1$. (I.e. there is a one-to-one correspondence from $V_1$ onto $V_2$.)

(b) Show Tutte’s 5-flow conjecture is true for cubic bipartite graphs.

2. Let $G$ be a simple graph that is not a null graph. Prove that the sum of the coefficients of $P_G(k)$ is 0. (Hint: When a function is a polynomial, how can one obtain the sum of the coefficients?)

3. The wheel graph $W_n$ is the $n$-vertex graph consisting of a cycle with $n - 1$ vertices and an additional vertex that is adjacent to all of the vertices in the cycle. The wheel graphs $W_4$, $W_5$, and $W_6$ are shown below:

Determine the chromatic polynomial of $W_n$ for $n \geq 4$. Prove your answer.

4. An infinite graph is a graph $G$ where both the vertex set $V(G)$ and the edge set $E(G)$ are infinite. (In Edition 4 of the textbook, see Section 16 for more information about infinite graphs. In Edition 5 of the textbook, see pages 24, 38, and 44.)

(a) Find an Eulerian trail in the infinite square lattice. (The infinite square lattice is Figure 16.1 in Edition 4 and Figure 1.45 in Edition 5 of the textbook.)

(b) Give an example of a connected infinite graph in which every vertex has even degree, but the graph is not Eulerian.