Math 142 **Midterm** **10/23/2015**

Name: ______________________________

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Always, Sometimes, Never questions. For all other questions, please circle your answers and show your work for full credit. There are 10 questions for a total of 100 points.

Always, Sometimes, Never: Read the statement and decide whether they are ALWAYS true, SOMETIMES true, or NEVER true.

_______ 1. (5 points) $\lim_{n\to\infty} \sum_{i=0}^{n} x_i^{-1} \Delta x = \log(2)$.

A. Always B. Sometimes C. Never

_______ 2. (5 points) Let $c \geqslant 4$. If $\int_1^c f(x)dx = 10$ and $\int_4^c f(x)dx = 4$, then $\int_1^4 f(x)dx = 14$.

A. Always B. Sometimes C. Never

_______ 3. (5 points) Suppose $f \geqslant 0$, f is continuous on $[a, b]$, and $\int_a^b f(x)dx = 0$. Then $f(x) = 0$ for all x in $[a, b]$.

A. Always B. Sometimes C. Never

_______ 4. (5 points) If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b x\sin(x)\ dx\right) = x\sin(x).$$

A. Always B. Sometimes C. Never

_______ 5. (5 points) If $f(x)$ is a continuous function on $[a, b]$, then $f(x)$ has an algebraic expression as an anti-derivative.

A. Always B. Sometimes C. Never

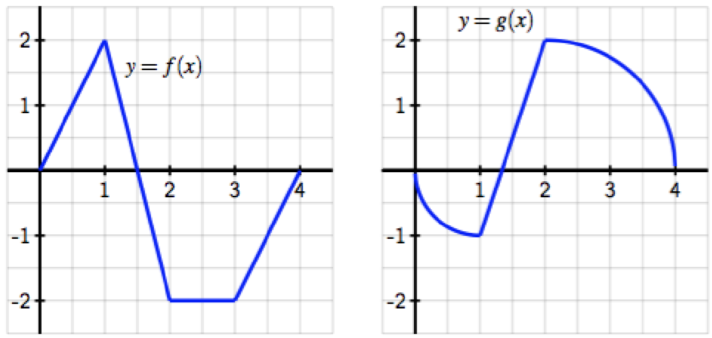
Short Answer. Make sure and justify your answer for full credit.

6. (20 points) Let $f(x)$ be a differentiable function such that:

$$\int_0^2 x^2 f'(x)dx = 4 \text{ and } \int_0^2 xf(x)dx = 6.$$

Find $f(2)$.

7. Consider the functions f and g as defined by the following graphs.



(a) (10 points) For what constant c does the following equation hold?

$$\int_0^1 c\ dx = \int_0^1 f(x) + g(x)dx$$

(b) (5 points) Let $A(x) = \int_0^x f(x)dx$. Exactly calculate $A(3)$ and $A(4)$.

8. (a) (10 points) Let $F(x)$ be defined to be the area under the curve $y = e^{t^2}$ between $t = 0$ and $t = x$. Find $F'(x)$.

(b) (5 points) Find $\frac{d}{dx}(F(e^x))$. (Hint: This is not a typo)

9. (a) (5 points) Determine the partial fraction decomposition of the following.

$$\frac{1}{(3x-2)(2x+5)}$$

(b) (5 points) Determine $\int \frac{1}{(3x-2)(2x+5)} dx$.

10. For an unknown function $f(x)$, the following information is known.

- f is continuous on $[3,6]$;
- f is either always increasing or always decreasing on $[3,6]$;
- f has the same concavity throughout the interval $[3,6]$;
- As approximations to $\int_3^6 f(x)\,dx$, $L_4 = 7.23$, $R_4 = 6.75$, and $M_4 = 7.05$.

(a) (5 points) Is f increasing or decreasing on $[3,6]$? What data gives you this information?

(b) (5 points) Is f concave up or concave down on $[3,6]$? Why?

(c) (5 points) Determine the best possible estimate you can for $\int_3^6 f(x)\,dx$, based on the given information.