Summary of Convergence and Divergence Tests for Series

Test	Series	Convergence or Divergence	Comments
n^{th}-term	$\sum a_n$	Diverges if $\lim\limits_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim\limits_{n\to\infty} a_n = 0$
Geometric Series	$\sum\limits_{n=1}^{\infty} ar^{n-1}$	(i) Converges with sum $S = \dfrac{a}{1-r}$ if $\lvert r\rvert < 1$ (ii) Diverges if $\lvert r\rvert \geq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to ar^{n-1}
p-series	$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^p}$	(i) Converges if $p > 1$ (ii) Diverges if $p \leq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to $\dfrac{1}{n^p}$
Integral	$\sum\limits_{n=1}^{\infty} a_n$ $a_n = f(n)$	(i) Converges if $\int_1^\infty f(x)dx$ converges (ii) Diverges if $\int_1^\infty f(x)dx$ diverges	The function f obtained from $a_n = f(n)$ must be continuous, positive, decreasing, integrable.
Comparison	$\sum a_n$, $\sum b_n$ $a_n > 0, b_n > 0$	(i) if $\sum b_n$ converges and $a_n \leq b_n$ for every n, then $\sum a_n$ converges. (ii) If $\sum b_n$ diverges and $a_n \geq b_n$ for every n, then $\sum a_n$ diverges. (iii) If $\lim_{n\to\infty}(a_n/b_n) = c$ for some positive real number c, then both series converge or both diverge	The comparison series $\sum b_n$ is often a geometric series or a p-series. To find b_n in (iii) consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum a_n$	If $\lim\limits_{n\to\infty} \lvert\dfrac{a_{n+1}}{a_n}\rvert = L$, the series (i) converges (absolutely) if $L < 1$ (ii) diverges is $L > 1 (or\ \infty)$	Inconclusive if $L = 1$. Useful of a_n involves factorials or n^{th} powers. If $a_n > 0$ for every n, disregard the absolute value.
Root	$\sum a_n$	If $\lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L, (or\ \infty)$, the series (i)converges (absolutely) if $L < 1$ (ii) diverges is $L > 1 (or\ \infty)$	Inconclusive if $L = 1$ Useful if a_n involves n^{th} powers If $a_n > 0$ for every n, disregard the absolute value.
Alternating Series	$\sum (-1)^n a_n$ $a_n > 0$	Converges if $a_k \geq a_{k+1}$ for every k and $\lim\limits_{n\to\infty} a_n = 0$	Applicable only to an alternating series.
$\sum \lvert a_n\rvert$	$\sum a_n$	If $\sum \lvert a_n\rvert$ converges, then $\sum a_n$ converges.	Useful for series that contain both positive and negative terms.