Math 142 **Exam 3** **04/15/2015**

Name: ______________________________

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the True/False or the Multiple Choice questions. For all other questions, please circle your answers and show your work for full credit. There are 15 questions for a total of 100 points.

True or False: Please circle either true or false. No work is necessary.

_______ 1. (5 points) If $\{a_n\}$ is a decreasing sequence and $a_n > 0$ for all n, then $\{a_n\}$ is convergent.

A. True B. False

_______ 2. (5 points) If $f(x) = 2(x-1) - (x-1)^2 + \frac{1}{3}(x-1)^3 - \cdots$ is convergent for all values of x, then $f'''(1) = 3$.

A. True B. False

_______ 3. (5 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$.

A. True B. False

_______ 4. (5 points) If the series $\sum c_n x^n$ diverges when $x = 6$, then the series diverges when $x = -10$.

A. True B. False

Multiple Choice: Please circle your answer. No work is necessary, but partial credit will be given if work is shown.

_______ 5. (5 points) If the limit of the sequence a_n defined by $a_{n+1} = -\dfrac{4}{4+a_n}$ exists, then the limit is

A. 1

B. -1

C. 2

D. -2

E. π

_______ 6. (5 points) Which of the following series are divergent? (There might be more than one.)

A. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+4n-1}{\sqrt{n^5+\pi n+9}}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^4}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\pi^n}$

D. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n+1}{e^{2n}}$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

_______ 7. (5 points) The radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n}$ is

A. 0

B. 5

C. ∞

D. $\frac{1}{5}$

_______ 8. (5 points) The Taylor series of $\sin(x^2)$ centered at $a=0$ is

A. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n+1)!}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

C. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$

D. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$

_______ 9. (5 points) Let $T_3(x)$ be the degree 3 taylor polynomial of $\sin(x)$ at $a = 0$. Using Taylor's inequality, the bound for

$$|R_3(x)| = |\sin(x) - T_3(x)|$$

for $x \in [0, 0.1]$ is

A. $\frac{1}{3!}(0.1)^3$ B. $\frac{x^4}{3!}$ C. $\frac{1}{4!}(x)^4$ D. $\frac{1}{4!}(0.1)^4$

_______ 10. (5 points) $\sum_{n=1}^{\infty} 2^{2n} 5^{1-n}$ is

A. convergent and equal to 5
B. convergent and equal to 5/4
C. convergent and equal to 20
D. divergent and equal to ∞
E. none of the above

_______ 11. (5 points) Which of the following statements are correct?

I. Every convergent series is absolutely convergent.
II. If a series is absolutely convergent, then it is convergent.
III. The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ is absolutely convergent.

A. I,II and III B. I and III C. II and III D. only III E. I and II

Short Answer: Show your work for full credit.

12. (5 points) Use series to evaluate the limit $\lim_{x \to 0} \dfrac{\sin(x) - x}{x^3}$.

13. (a) (5 points) Explain why the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ is convergent.

(b) (5 points) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ to two decimal places.

14. (15 points) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$.

15. (a) (10 points) Find the power series representation of $\dfrac{1}{(1-x)^2}$ (HINT: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{1-x}\right) = \dfrac{-1}{(1-x)^2}$).

(b) (5 points) What is the radius of convergence of the series in part (a)?