Math 142   Exam 2   03/06/2015

Name: _____

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the True/False questions. For all other questions, please circle your answers and show your work for full credit. There are 10 questions for a total of 100 points.

True or False: Please circle either true or false. No work is necessary.

_____ 1. (5 points) $\dfrac{d}{dx}\left(\displaystyle\int_{4}^{12} f(x)\ dx\right) = f(x)$.

          A. True   B. False

_____ 2. (5 points) If f is continous, then $\displaystyle\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to\infty}\int_{-t}^{t} f(x)dx$.

          A. True   B. False

_____ 3. (5 points) Using the midpoint rule to approximate $\displaystyle\int_{-1}^{1}\cos x\ dx$ within an error of 0.001, the minimum number of partitions to use is 10.

          A. True   B. False

_____ 4. (5 points) Let $L = \displaystyle\int_{1}^{2} t\sqrt{4 + 9t}\ dt$. Then L is the length of the curve $x = t^2$, $y = t^3$ that lies between $(1, 1)$ and $(4, 8)$.

          A. True   B. False

_____ 5. (5 points) The Midpoint Rule is always more accurate than the Trapezoidal Rule.

          A. True   B. False

Short Answer. Make sure and justify your answer for full credit.

6. (a) (5 points) Find $\displaystyle\int \frac{5x-4}{2x^2+x-1}dx$.

   (b) (5 points) Find $\displaystyle\int_4^\infty e^{-y/2}dy$.

(c) (5 points) Find the average value of $f(x) = \log x$ on the interval $[1, 3]$.

7. (15 points) Suppose that f is continuous on $[0, \infty)$ and that $\lim_{x \to \infty} f(x) = 1$. Is it possible that $\int_0^\infty f(x)dx$ is convergent? Fully justify your answer.

8. (15 points) Use Simpson's Rule with $n = 6$ to estimate the volume obtained by rotating about the y-axis the region under the curve $y = e^{x^2}$, $0 \leqslant x \leqslant 1$. What is the error bound?

9. (15 points) If f is continuous and $\int_4^8 f(x)dx = 12$, is it possible that f takes on the value 3 at least once on the interval $[4, 8]$? Fully justify your answer.

10. (15 points) A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m (Note that the base radius is the top of the tank, it's inverted. Think waffle cone.). It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3.)