In this talk we will see a brief overview of Hilbert functions, a very powerful and well established tool in Algebra, Combinatorics, and in many other fields of Mathematics, even in more applied ones. We will go through some computations, with particular emphasis on the relations with singularities of curves: for example, the cuspidal curve $y^2 = x^3$ has a singular point (a cusp) at the origin, while the parabola $y = x^2$ is a nice and smooth curve. We will see how the Hilbert function detects the difference between these two objects. We will also briefly discuss how to use Hilbert functions as a tool for counting vertices, edges and faces of a simplicial complex (essentially, a generalized version of a triangle). As a more practical application, we will see how to determine all possible ways of giving change for a dollar.