Math Teachers' Program 621
Introduction to Mathematical Logic

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1 Subject Matter

Logic is a way to come to know a fact without observing it directly. If your roommate always gets drunk on Saturday nights, and today was Saturday, you don't have to smell your roommate's breath to know that (s)he is drunk. There are, of course, many more subtle applications of logic, and frequently it's not so obvious what conclusions you can draw from what. The careful study of this question, and the field of logic itself, dates approximately to Aristotle. Aristotle's treatment of logic was so widely respected that the field changed little for two thousand years, until the 19th century when mathematicians like Boole, Frege, Peano, and Peirce cleaned it up to roughly its present state.

Since logic, like other branches of mathematics, has trouble dealing with ambiguous statements, and since English is a notoriously ambiguous language, logicians (and other mathematicians) usually work in a language of their own, in which every term and symbol has a clear, well-defined meaning — the "language of first-order logic".

2 Student Outcomes

The official bulletin description of this course says

> Logical paradoxes and controversies. Truth functions and quantifiers. Naive set theory. Axiomatization of first-order logic. Deductive theories.

But that's not really a good description of what we'll be talking about this semester. Since everyone in this course is planning to be a high school math teacher, we'll concentrate on topics of use in that setting. We'll look fairly intensively at first-order predicate logic, with its syntax, semantics, and proof techniques. For every common first-order logical operator ($\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \forall, \exists$), we'll discuss the meaning of truth in a given model for statements constructed that way, as well as techniques for proving and refuting such statements. Students should end the semester with a solid grasp of these techniques, as well as applications of them such as mathematical induction on natural numbers and on sets. We'll end the semester with a few advanced topics which may, nonetheless, be useful in high school-level projects: resolution and the Prolog programming language, completeness and incompleteness, computability and uncomputability, etc. depending on time.

3 Textbook

We'll use the textbook *Language Proof and Logic*, by Jon Barwise and John Etchemendy, ISBN 1-57586-374-X, and will work through most of it by the end of the semester: certainly chapters 1-13, 16, and possibly some others. *You are responsible for everything in the reading assignments, whether or not I discuss it in a lecture.*

The textbook comes with several software applications: "Tarski's World", for understanding the semantics of first-order logic sentences; "Boole", for building and evaluating truth tables; "Fitch", for writing and checking formal proofs; and "Submit", an auto-grading system for exercises written using the other three. If you buy a used copy of the textbook, "Submit" may not work for you, but the other three should be OK, and that's enough to complete the course.

4 Grading

I expect to have about five homework assignments, each worth 15% of the semester grade, and a final exam worth 25%. I may change this somewhat as the semester unfolds.

The exam must be taken at the scheduled time, unless arranged in advance or prevented by a documented medical or family emergency. If you have three or more exams scheduled on the same date, or a religious holiday that conflicts with an exam or assignment due date, please notify me in writing within the first two weeks of the semester in order to receive due consideration. Exams not taken without one of the above excuses will be recorded with a grade of 0.

Homework assignments will be accepted late, with a penalty of 10% per 24 hours or portion thereof after they're due. An hour late is 10% off, 25 hours late is 20% off, and so on. I want you to turn things in on time, but since each assignment may build on previous ones, it's better to do an assignment late than never.

5 Ethics

Most of the assignments in this class are to be done individually. You may *discuss general approaches* to a problem with classmates, but you *may not copy* large pieces of homework solutions. If you do, *all* the students involved will be penalized. If I choose to allow a particular assignment to be done in groups, I'll tell you so clearly.

All work on an exam must be entirely the work of the one person whose name is at the top of the page. If I have evidence that one student copied from another on an exam, *both* students will be penalized; see above.

6 Schedule

This class meets every Tuesday from 6:30-8:20 PM, except on University holidays or if I cancel class. The schedule of topics, readings, and homework assignments is on the Web at
`http://home.adelphi.edu/sbloch/class/621/calendar.shtml`
Check it from time to time. All dates in the schedule are tentative, except those fixed by the University; if some topic listed here as taking one lecture in fact takes two lectures to cover adequately, or *vice versa*, the schedule will shift.

I expect you to have read the specified chapters in the textbook before the lecture that deals with that topic; this way I can concentrate my time on answering questions and clarifying subtle or difficult points in the textbook, rather than on reading the textbook to you, which will bore both of us. **Please read the textbook!**